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Optimization of the cold supply chain logistics network with an environmental dimension

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Abstract. This article is the result of generalizing a real practical problem of optimizing the operation of a cold supply logistics network. A small (up to 100 clients) logistics company solves the problem of operational planning: during the working day (fixed time window) to provide the supply of cold (food) products from distribution centers to end consumers along well-defined routes (i.e., the logistic network graph is fixed). The company has set the following conditions for the authors. In addition to minimizing the usual transport costs, take into account also the environmental costs (associated with negative consequences for the environment) that arise during the operation of the logistics supply network. And also when deciding to use software available to the company. The case is considered when the transport tariff on each route is a linearly increasing function of the total volume of all traffic on this route. When the Nash equilibrium conditions are met, the mathematical model for optimizing the total costs of the logistics network is reduced to a convex (quadratic) programming problem. Computer implementation in MS Excel environment has been carried out. This makes it possible to carry out simulation calculations for various options for transport costs on routes, taking into account environmental costs.

1. Introduction

As you know, modeling a logistics network consists of two interrelated processes: designing the network and optimizing operation. Logistics network design (logistics design) is the process of selecting and evaluating the configuration, which minimizes overall costs and improves the performance of each link in the supply chain. This process solves two main problems of sustainable logistics: the issue of the location of distribution centers (depots), as well as the problem of the flow of products (goods) passing through the network from suppliers to consumers. The main components of all mathematical models for the design of supply chains are the sets of nodes and directed arcs (routes) that are available for network design. To find optimal solutions, various fixed and variable costs (capital, operating, transport, penalty and others) are included in mathematical models, and then, as a rule, the total costs of the entire logistics network are minimized. The dynamism and diversity of information in logistics networks lead to different (unique for each specific network) tasks of designing their structure and optimizing the operation of supply chains. Therefore, mathematical models of various logistics networks of supply chains can differ significantly, having their own specific features (see, for example, the review [1]).

The classical vehicle routing problem (The Vehicle Routing Problem, VRP) was first formulated in 1959 by G B Dantzig, J H Ramser. This problem is a generalization of the well-known NP-hard Traveling Salesman Problem (TSP), for which methods and algorithms are still unknown that allow finding exact or approximate solutions with a given error estimate in polynomial time. Therefore, it is now believed that for most varieties of the VRP problem and the general TSP problem there is no

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polynomial and, moreover, a completely polynomial approximation scheme (Polynomial and Fully Polynomial Time Approximation scheme, PTAS and FPTAS. In 1964, Clark and Wright (G. Clarke and JW Wright) was the first to propose an approximate algorithm for solving the VRP problem. For problems with a small number of clients ($n \le 135$), precise algorithms based on dynamic programming techniques have been developed. Later, intensive development of methods and algorithms for solving the VRP problem and its generalizations began. In recent years, various evolutionary (heuristic) algorithms have been actively used, which for solving some individual problems are more effective than classical ones (see, for example, [2-7]). It should be noted that evolutionary algorithms do not have a rigorous justification, since they usually look for local extrema (which may not coincide with global ones). The operation of the logistics supply network inevitably leads to negative environmental consequences: various pollution, emissions of gases harmful to the environment.

Therefore, in the mathematical models of the logistics network, in addition to the usual costs, they began to include environmental costs (either as an integral part of the total costs in the target function, or as all kinds of environmental restrictions) to minimize the overall negative impact on the environment. The development of models and methods for solving VRP problems with environmental components is given close attention in modern research ([8-16]).

2. Formulation of the problem

Some logistics company is engaged in the supply of cold products (in our case, food), which requires transportation by refrigerated trucks. m distribution points (depots) are set, which are interconnected by n arcs (routes) along which goods are delivered. A portfolio of L orders from customers - end consumers (retailers) is set, for each of which the point of departure and the point of delivery, as well as the order volume, are known. All goods from the trunk hubs must be delivered to customers (i.e., to meet their demand) within an eight-hour working day. Thus, a fixed closed logistics network of cold supplies with a fixed time window is considered.

3. Mathematical model

Let us denote $\mathbf{d}^l \in R^m$ - vectors, the components of which $\mathrm{d}_i^l, i=\overline{1,m}; l=\overline{1,L};$ are equal to the volumes of delivery from the depot i to the client l. Note that if some components ($\mathrm{d}_i^l < 0$) of the vectors $\mathbf{d}^l \in R^m$, are negative, the value $\left|\mathrm{d}_i^l\right|$ corresponds to the volume of delivery from the client l to the depot i. Since a closed logistics network is considered (the volumes of goods that enter the logistics network and are delivered to all customers are the same), then the balance condition is satisfied for the vectors $\mathbf{d}^l$: $\sum_{i=1}^{m} \mathrm{d}_i^l = 0, l=\overline{1,L};$. For the graph corresponding to the given logistic network, a matrix $\mathbf{A} = \left(a_{ij}\right), i=\overline{1,m}; j=\overline{1,\mathrm{n}};$ of incidences of depots and routes is constructed, the elements of which are numbers: $a_{ij} = \begin{cases} 1, & \textit{if the route } j \textit{ enter the node } i \\ -1, & \textit{if the route } j \textit{ leaves the node } i \\ 0, & \textit{if the route } j \textit{ is not connected to the node } i \end{cases}.$

By denote the $\mathbf{x}^l \in R^n$ vector whose components are equal to the l-th customer's transportation volumes along the route j. This vector will be a valid customer l transportation plan if the following conditions are met:

$$\mathbf{A}\mathbf{x}^\mathbf{l} = \mathbf{d}^\mathbf{l}, \quad \mathbf{x}^\mathbf{l} \ge \mathbf{0}. \tag{1}$$

$$\sum_{i=1}^{m} \mathrm{d}_i^l = 0, l=\overline{1,L}. \tag{2}$$

The transportation plan of all customers is a matrix $\mathbf{X}_{n\times L} = \begin{bmatrix} \mathbf{x}^1 & \mathbf{x}^2 & \cdots & \mathbf{x}^L \end{bmatrix}$, the columns of which are vectors $\mathbf{x}^l \in R^n$ $(l = 1, 2, ..., L)$ of customer transportation plans. This shared transportation plan for all customers will be valid if each customer's transportation plans are valid. We denote by $\mathbf{s} \in R^n$ - a vector, the components of which s_j are equal to the total volume of all traffic on a given separate an route j, i.e.:

$$s_j = \sum_{l=1}^{L} x_j^l; \quad j = 1, 2, ..., n;. \tag{3}$$

The tariff for transportation along an route j is considered as a linearly increasing function of the total volume s_j of traffic of all customers along this route:

$$\mathrm{T}_j(\mathrm{s}_j) = e_j\,\mathrm{s}_j + c_j,\ j = \overline{1,n};\ , \tag{4}$$

where $e_j = e_j^1 + e_j^2,\ j = \overline{1,n};$ are the coefficients that express environmental costs (in our case, the cold supply logistics network e_j^1, e_j^2 is the costs associated with CO2 emissions from car engines and refrigeration units, respectively), and c_j are the usual transportation costs. The validity of the introduction of such tariffs is confirmed by numerous empirical studies. As an example, we present a graph following from [17, 18], which demonstrates the quadratic relationship between emission reductions and an increase in total costs:

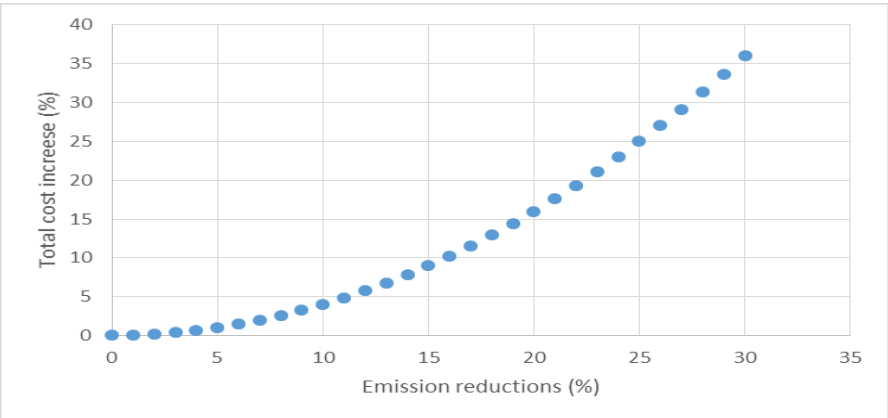

Figure 1. Graph of the relationship between emission reductions and corresponding increases in total costs.

It is obvious that the coefficients e_j^1, e_j^2 characterizing the environmental costs depend on the types of vehicles and refrigerators used, traffic patterns, road quality, emission taxes and other factors. As a rule, they are calculated based on empirical data (see, for example, [11-13, 17, 18]). Let's form the target function for each individual client. The value of the l-th client's costs for transportation along the route j (as the product of the tariff and the volume of traffic) has the form of a function of the total traffic s_j and the client's transportation along the route:

$$F_j^l\left(s_j, x_j^l\right) = T_j\left(s_j\right)\cdot x_j^l. \tag{5}$$

Summing up the costs for all routes, we obtain the total costs of the customer's transportation as a function of the transportation plan $\mathbf{x}^l$ of the given customer and the vector $\mathbf{s}$, the components of which s_j are equal to the total transportation of all customers along all routes:

$$\mathrm{F}^l(\mathbf{s}\ ,\mathbf{x}^l) = \sum_{j=1}^{n} \mathrm{F}_j^l(\mathrm{s}_j, \mathrm{x}_j^l)\,. \tag{6}$$

Each client, when choosing his optimal transportation plan, solves the problem of minimizing his transportation costs:

$$\mathrm{F}^l(\mathbf{s}\ ,\mathbf{x}^l) \to \min \tag{7}$$

with restrictions (1): $\mathbf{A}\mathbf{x}^l = \mathbf{d}^l, \quad \mathbf{x}^l \geq \mathbf{0}$.

In this form, the multicriteria task of finding a general optimal transportation plan is not fully defined, since the vector of the transportation plan of an individual client influences, according to (3), the vector, that is, the choice of all customers. Therefore, let us define the mathematical model of our problem with the Nash equilibrium condition, which for our problem means that it is not profitable for any client to change his plan. Let us formulate the Nash equilibrium condition:

An admissible general plan of transportation of all customers $\overline{\mathbf{X}} = \begin{bmatrix} \overline{\mathbf{x}}^1 & \overline{\mathbf{x}}^2 & \cdots & \overline{\mathbf{x}}^L \end{bmatrix}$ will be Nash equilibrium if for no $l = 1, 2, ..., L;$ there is no valid plan $\mathbf{X} = \begin{bmatrix} \mathbf{x}^1 & \mathbf{x}^2 & \cdots & \mathbf{x}^L \end{bmatrix}$ such that $\mathbf{x}^k = \overline{\mathbf{x}}^k, k \in \{1, 2, ..., L\}, k \neq l;$ and at the same time $\mathrm{F}^l(\tilde{\mathbf{s}}, \mathbf{x}^l) < \mathrm{F}^l(\overline{\mathbf{s}}, \overline{\mathbf{x}}^l)$, where $\tilde{\mathbf{s}} = \sum_{m=1}^{L} \mathbf{x}^m, \quad \overline{\mathbf{s}} = \sum_{m=1}^{L} \overline{\mathbf{x}}^m$.

The transition from plan $\overline{\mathbf{X}}$ to plan $\mathbf{X}$ is carried out only by changing the plan of one of the clients, while the transportation plans of the other clients remain unchanged. And during the transition from the plan $\overline{\mathbf{x}}^l$ of the l-th client to the plan $\mathbf{x}^l$, the effect of this transition on tariffs due to changes in the vectors of total traffic along the routes is taken into account.

In [19], it was proved that in this case, under the condition of consistency of constraints (1), the Nash equilibrium exists and is unique (since all conditions of the Kuhn-Tucker theorem are satisfied). Finding a Nash equilibrium leads to the following quadratic programming problem:

Find variables (vector components $\mathbf{x}^l$, $\mathbf{u}^l$; $l = 1,2,...,L;$) that minimize the objective function of total transport costs (including environmental costs)

$$Y(\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^L) - \Lambda(\mathbf{u}^1, \mathbf{u}^2, ..., \mathbf{u}^L) \to \min \tag{8}$$

with restrictions

$$y_j^l(x_j^1, x_j^2, ..., x_j^L, \mathbf{u}^l) \geq 0, \tag{9}$$

$$x_j^l \geq 0, \quad j = 1,2,...,n; l = 1,2,...,L; \tag{10}$$

where $$Y(\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^L) = \frac{\sum_{j=1}^{n} e_j \cdot \left(\left[\sum_{i=1}^{L} x_j^l \right]^2 + \sum (x_j^l)^2 \right)}{2};$$

$$\Lambda(\mathbf{u}^1, \mathbf{u}^2, ..., \mathbf{u}^L) = \sum_{l=1}^{L} \sum_{i=1}^{n} d_i^l u_i^l;$$

$\mathbf{u}^l$ are the vectors of Lagrange multipliers of constraints (1) of problem (7), and

$$y_j^l(x_j^1, x_j^2, ..., x_j^L, \mathbf{u}^l) =$$
$$= e_j x_j^l + e_j \sum_{k=1}^{L} x_j^k - \left(\mathbf{A}^T \mathbf{u}^l\right)_j + c_j; \quad j = \overline{1,n}; l = \overline{1,L}.$$

Thus, the mathematical model for a given logistics network of cold supply chains, taking into account environmental costs, is the problem of minimizing a quadratic convex function (8) under constraints (9), (10). Problems of this type have a unique solution (global minimum) that can be found by classical methods.

4. Computer implementation

Computer implementation was carried out in MS Excel. For simulation calculations, a macros has been developed in Visual Basic for Application, which prepares an objective function for the Solver add-in and limits a specific logistic problem. The following are set: the structure of the logistics network of cold supply chains, customer demand, transport costs on delivery routes. To find the optimal designs, the Generalized Reduced Gradient method was used to solve nonlinear optimization problems of the Solver add-in. Simulation calculations were made with different environmental (depending on different taxation rates for CO2 emissions) and constant usual transport costs on the routes. This made it possible to assess the contribution to the total costs of operating the logistics network of environmental costs (and thereby the degree of environmental impact). Let's consider a demo example (conditional data). The structure of the logistics network of cold supply chains is defined by the network graph (Figure 2), and the available routes – by the incidence matrix of the graph vertices:

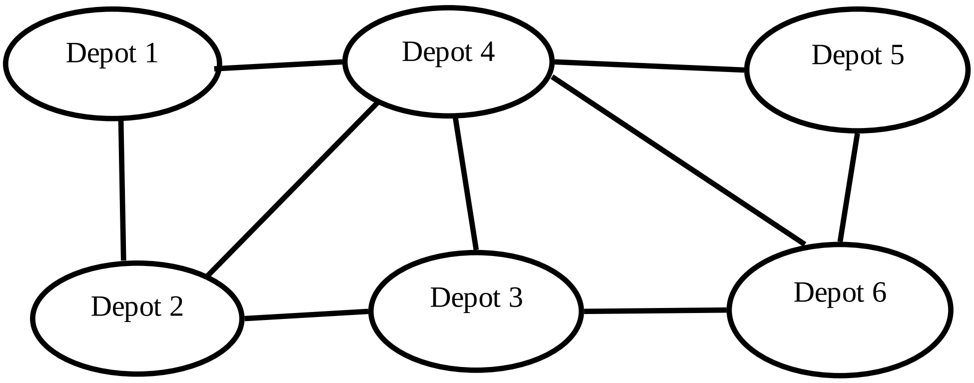

Figure 2. Graph of the logistics network.

In accordance with the specified logistic network of cold supply routes (Figure 2) and the number of customer-consumers, an incidence matrix A of distribution points (Depots) and routes is formed.

Customer demand is shown by the table 1.

Table 1. Volumes of customer demand - consumers of cold products.

Consumers	Depot 1	Depot 2	Depot 3	Depot 4	Depot 5	Depot 6
1	20	0	0	0	0	-20
2	26	0	0	0	0	-26
3	0	40	0	0	0	-40
4	0	0	0	100	0	-100
5	0	0	0	0	-20	20
6	0	0	39	0	0	-39
7	0	0	0	0	3	-3
8	0	0	58	0	0	-58

On routes, coefficients e_j are set that characterize environmental costs, and c_j the usual transport costs. Then a special VBA macro prepares the objective function, constraints, initial plan for the Solver program and starts the Solver program using the Generalized Reduced Gradient Method. The result of the Solver program is the General optimal plan for the distribution of supplies along the logistics network and the minimum total transport costs 119152.8 (currency units).

5. Conclusion

A mathematical model has been developed for a fixed logistics network of cold supply chains (refrigerated trucks), taking into account environmental costs. The time window is also fixed (8 hour working day). Under the condition of the Nash equilibrium, the problem is reduced to a quadratic programming problem (minimization of a convex quadratic function under linear constraints). The obtained results (taking into account the developed computer implementation, which allows for simulation calculations) can be applied in practice by small logistics companies dealing with cold supplies. At the same time, the company minimizes not only the usual transport costs, but also the costs associated with harmful effects on the environment.

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