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MATHEMATICAL MODEL CONSTRUCTION OF TRANSPORT SYSTEMS OPERATION PROCESSES USING FUZZY LOGIC METHODS

Abstract. *The article emphasizes that the improvement of management, finding and using of new methods of work optimization on the routes of urban passenger transport are the main tasks of organizing route transportation in the considered transport systems. The search for these methods is aimed at the rational use of resources, vehicles, to meet the needs of the population in travel and improve the quality of service in the implementation of passenger traffic. To ensure optimal management of work on the route of urban passenger transport, we will take as a goal the construction of a timetable for the movement of vehicles on the route, namely the creation of schedules for the release of transport and rational organization of work. The creation of these schedules takes into account the effective using use of the car fleet of enterprises, ensuring the safety of passenger transportation, and reducing the cost of services for the population. In this article, the study of passenger traffic on a certain section of the transport network and the operation of the route on it are considered using fuzzy logical methods, which gives the result of the consideration of service time minimization and maximum level of passenger satisfaction with the use of transport. In this case, the mechanism of working with inaccurate concepts in the process of finding a solution is implementing, we can imagine the input data as fuzzy sets on which logical actions are carried out. In this study, the numerical expression of the degree of satisfaction of passenger's needs is chosen by the function of belonging to several time intervals, which is selected by the target function of finding the optimal schedule in these time intervals. The search for a solution takes into account the system of restrictions that apply to the time intervals of vehicles using. At the same time, the criterion of optimality reflects the cost of the schedule working, which includes the cost of operating time of vehicles and the cost of waiting time for passengers when traveling along the route.*

Keywords: *mathematical modeling; algorithmization; optimization methods; transport systems; passenger transportation.*

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ПОБУДОВА МАТЕМАТИЧНОЇ МОДЕЛІ ПРОЦЕСІВ РОБОТИ ТРАНСПОРТНИХ СИСТЕМ З ВИКОРИСТАННЯМ МЕТОДІВ НЕЧІТКОЇ ЛОГІКИ

Анотація. У статті наголошено на тому, що удосконалення управління, перебування та використання нових методів оптимізації роботи на маршрутах міського пасажирського транспорту є основними завданнями організації маршрутних перевезень у розглянутих транспортних системах. Пошук цих методів спрямований на раціональне використання ресурсів, транспортних засобів, на задоволення потреб населення у пересуванні та удосконалення якості обслуговування при здійсненні пасажирських перевезень. Для здійснення оптимального управління роботою на маршруті міського пасажирського транспорту візьмемо в якості мети побудову розкладу руху транспортних засобів на маршруті, а саме створення графіків випуску транспорту та раціональної організації роботи. При створенні цих розкладів враховується ефективне використання автомобільного парку підприємств, забезпечення безпеки перевезень пасажирів, зниження вартості послуг для населення. У цій статті вивчення пасажиропотоків на певній ділянці транспортної мережі та робота маршруту на ній розглядаються за допомогою нечітких логічних методів, що дає в результаті розгляду мінімізацію часу обслуговування та максимальний рівень задоволеності пасажирів використанням транспорту. При цьому реалізується механізм роботи з неточними поняттями в процесі пошуку рішення, вхідні дані ми можемо уявити як нечіткі безлічі, над якими здійснюються логічні дії. У даному дослідженні для чисельного вираження ступеня задоволення потреб пасажирів обирається функція приналежності до декількох тимчасових інтервалів, яка вибирається цільовою функцією пошуку оптимального розкладу руху в дані тимчасові інтервали. При пошуку рішення враховується система обмежень, що застосовуються до тимчасових інтервалів використання транспортних засобів. При цьому критерій оптимальності відображає вартість роботи графіка, що включає вартість часу експлуатації транспортних засобів і вартості часу очікування пасажирів при пересуванні за маршрутом.

Ключові слова: математичне моделювання; алгоритмізація; методи оптимізації; транспортні системи; пасажирські перевезення.

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ПОСТРОЕНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ ПРОЦЕССОВ РАБОТЫ ТРАНСПОРТНЫХ СИСТЕМ С ИСПОЛЬЗОВАНИЕМ МЕТОДОВ НЕЧЕТКОЙ ЛОГИКИ

Аннотация. *В статье отмечено, что усовершенствование управления, поиск и использование новых методов оптимизации работы на маршрутах городского пассажирского транспорта являются основными задачами организации маршрутных перевозок в рассматриваемых транспортных системах. Поиск этих методов направлен на рациональное использование ресурсов, транспортных средств, на удовлетворение потребностей населения в передвижении и усовершенствование качества обслуживания при осуществлении пассажирских перевозок. Для осуществления оптимального управления работой на маршруте городского пассажирского транспорта возьмем в качестве цели построение расписания движения транспортных средств на маршруте, а именно создание графиков выпуска транспорта и рациональной организации работы. При создании этих расписаний учитывается эффективное использование автомобильного парка предприятий, обеспечение безопасности перевозок пассажиров, снижение стоимости услуг для населения. В данной статье изучение пассажиропотоков на определенном участке транспортной сети и работа маршрута на нем рассматриваются с помощью нечетких логических методов, что дает в итоге рассмотрения минимизацию времени обслуживания и максимальный уровень удовлетворенности пассажиров использованием транспорта. При этом реализуется механизм работы с неточными понятиями в процессе поиска решения, входные данные мы можем представить как нечеткие множества, над которыми осуществляются логические действия. В данном исследовании для численного выражения степени удовлетворения потребностей пассажиров выбирается функция принадлежности к нескольким временным интервалам, которая выбирается целевой функцией поиска оптимального расписания движения в данные временные интервалы. При поиске решения учитывается система ограничений, применяемых к временным интервалам использования транспортных средств. При этом критерий оптимальности выражает стоимость работы графика, включающую стоимость времени эксплуатации транспортных средств и стоимости времени ожидания пассажиров при передвижении по маршруту.*

Ключевые слова: *математическое моделирование; алгоритмизация; методы оптимизации; транспортные системы; пассажирские перевозки.*

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1. Introduction

In considering different approaches and methods for improving the performance of passenger transport in cities, we will look at appropriate options for improving the management of passenger transport on routes. The following indicators can be used as such regulators: reducing the intervals between exits to the route for transportation by specific vehicles, developing schedules that take into account the improvement of the quality of work on the route, attracting other types of vehicles with a certain passenger capacity. Adjustments and creation of new route schedules, new passenger service

schedules for rolling stock vehicles should also be based on changes in traffic intervals on the route, on changes in the start and end times of passenger transport [2; 3].

It is of great importance in these developments to take into account changes in passenger traffic during working days on the route and to study the intensity of passenger traffic and the number of vehicles operating on the route [1; 4] built on the necessary volumes of traffic.

Thus, the construction of schedules based on a change in the time of operation of vehicles on the route depending on the number of passengers using bus passenger transport at certain periods of time and in certain sections is a promising direction for the development of methods of managing and regulating the work of routes of transport systems of cities.

Many scientific works have also recently provided the development of new effective methods and tools for the management of passenger traffic, which are aimed at minimizing the service time, achieving the maximum level of satisfaction of passengers in transport needs in the necessary, economically active sections of the city's transport network and minimizing costs when using vehicles on the route, that is, energy saving of urban economy resources [5–7].

2. Aim and methodology of research

The study of passenger traffic is useful to consider using fuzzy logic methods, which guarantee a high level of input data processing and a low delay in the processing time of this data. Also, fuzzy logic helps determine the validity of practical results. Fuzzy logic with correctly entered values also gives intermediate estimates of criteria between an absolutely correct value and an absolutely false value, taking into account the fact that with large values of the function of belonging to a certain set, the validity of the criterion in question is higher.

The process of applying fuzzy logic to input values is a complete number of rules that are used in obtaining initial values, and accordingly in making decisions to improve technical processes.

3. Literature review, shortcomings and problem statement

Many scientific works have also recently provided the development of new effective methods and tools for the management of passenger traffic, which are aimed at minimizing the service time, achieving the maximum level of satisfaction of passengers in transport needs in the necessary, economically active sections of the city's transport network and minimizing costs when using vehicles on the route, that is, energy saving of urban economy resources [5–7].

The concept of a fuzzy set was introduced by L. A. Zadeh in the works of 1965 [8], it is the concept of a set with fuzzy boundaries. To replace the objective function of holistic programming, a function of belonging to a set is introduced, the values of which vary in the range from 0 to 1, $\mu(x) \in [0; 1]$, indicating the degree of correspondence to the selected set. Enter the designation X of some set of values regarding which criterion A is discussed, then an odd set of elements X will be provided as a set of ordered pairs $\{(x, \mu_A(x) | x \in X\}$, $\mu_A(x) -$ a degree indicating how much x belongs to the fuzzy set A. Thanks to the concepts introduced in the further development of this theory,

R. E. Bellman in 1970 [9] the problems of integer programming were able to reduce to solving problems in the concepts of fuzzy logic, in which the objective function and the constraint system are written as fuzzy sets.

Solving these problems means solving the problem of compatible fulfillment of fuzzy goals and fuzzy restrictions and, that is, crossing the corresponding sets.Fuzzy sets and operations with them will be used to program and control processes in urban passenger transport. Let the fleet of a certain transport enterprise consist of vehicles of rolling stock in unlimited numbers, first make such an assumption. This transport enterprise serves a certain route of passenger urban transportation. Consider possible options for moving buses and boarding passengers on buses at stops.

The study of passenger traffic is useful to consider using fuzzy logic methods, which guarantee a high level of input data processing and a low delay in the processing time of this data. Also, fuzzy logic helps determine the validity of practical results. Fuzzy logic with correctly entered values also gives intermediate estimates of criteria between an absolutely correct value and an absolutely false value, taking into account the fact that with large values of the function of belonging to a certain set, the validity of the criterion in question is higher.

The process of applying fuzzy logic to input values is a complete number of rules that are used in obtaining initial values, and accordingly in making decisions to improve technical processes.

4. Presentation of the main material of the study

The concept of time states that create a space of time states.

As described in the works of R. E. Bellman [9], the space Ω of continuous service time turns into a discrete finished space of states Ω, where $|\Omega| < \infty$ with the same time intervals of 1 minute

$$\forall \omega_i, \omega_{i+1} \in \Omega: \ \omega_{i+1} - \omega_i = 1 \tag{1}$$

The concept of the space of time stages (time segments).

The space of the time stage K is divided into the space of the time stage Q, where $N = |Q| < \infty$ at equivalent intervals $s \in [5, 30]$ depending on the passenger service. The time state corresponds to time step $q(\omega, s)$, which is defined as follows:

$$q(\omega, s) = [\omega, s].$$

The schedule of buses on the passenger service route.

$Z = \{z_1, z_2, \dots, z_N\}$ – a schedule that represents and provides service intervals. This refers to the service of passengers on the route by a certain bus, at which z_N is the moment of departure on the route of the N-th bus, which is counted from the moment of exit of the first bus on the route. Also, the start and end times of passenger service on the route z_1 and z_N are set first, where N is the number of buses operating on the route.

Find the values of the variables that will solve the problem.

$t \in T$ are values belonging to the final set of all time interval values and are time intervals passing between two consecutive bus stops:

$$t_1 < t_2 < \cdots < t_M \, , |T| = M, \; t_{i+1} - t_i = 1, \forall i = \overline{1, M-1} \qquad (2)$$

A lot of stops on the route.

We will assume that the number of the passengers served at intervals of time t_i, $i = \overline{1, M}$ is evenly distributed. The number of stops is denoted by $b_j \in \{b_1, b_2, \dots, b_J\}, j = \overline{1, J}$.

Use fuzzy logic methods to create an optimal route schedule.

The function of belonging to a plurality of time slots corresponding to the fuzzy function of the target, which many times determines the degree of satisfaction of the needs of passengers, the number of which $K_j^{\omega,t}$ is the number of passengers sent from the stop j, will be indicated $\mu_{g,P}(K_j^{\omega,t})$, where P – is the maximum capacity of the vehicle.

Denote $\mu_g^{\omega}(t)$ and $\mu_c^{\omega}(t)$ the belonging functions that are used for the objective function and the constraint system, the argument of these functions is the time interval t, which is considered in the time state ω. Belonging of the time interval to the required set of time intervals is considered for each stop $j = \overline{1, J-1}$ considering the significance of this stop as the weight indicated above by equation (2).

For these membership functions, we obtain the following formulas:

$$\mu_g^{\omega}(t) = \sum_{j=1}^{J-1} b_j^q \cdot \mu_{g,P}(K_j^{\omega,t}), \mu_c^{\omega}(t) = \sum_{j=1}^{J-1} b_j^q \cdot \mu_{c,P}(K_j^{\omega,t}) \qquad (3)$$

Next, we write down an belonging function that indicates the degree of correspondence of the vehicle service time interval t to the fuzzy objective function and fuzzy restrictions, that is, the belonging function to the plurality of time intervals that most correspond to the fuzzy target and fuzzy restrictions in a certain time state ω. This formula will be as follows:

$$\mu_O^{\omega}(t) = \mu_g^{\omega}(t) \wedge \mu_c^{\omega}(t) \qquad (4)$$

In order to estimate the optimality of the traffic schedule Z of vehicles on the route, it is necessary to calculate the values that are characteristics of this schedule, and using the values of these characteristics it is possible to optimize the traffic schedule. First, we write down formulas for calculating the total time of service of passengers by vehicles on the route T_Z, the total time of movement of passengers depending on the state of the transport system T_ω, the total waiting time of passengers of vehicles on the route T_w:

$$T_z = \sum_{\omega=z_1}^{z_N} \sum_{j=1}^{J-1} \omega_j^q \,, \qquad T_\omega = \sum_{\omega=z_2}^{z_N} \sum_{j=1}^{J-1} (K_j^{\omega,t} \cdot \omega_j^q), \qquad T_w = \sum_{\omega=z_2}^{z_N} \sum_{j=1}^{J-1} \frac{v_j^q \cdot t_\omega^2}{2} \qquad (5)$$

Next, we can derive the formula for calculating the average degree of satisfaction of the population's needs for buses carrying out transportation on the route during the working

day as the average value of the function of belonging to the set of time intervals most corresponding to the fuzzy goal, that is, the degree of correspondence of the objective function. A formula will also be written to calculate the average value of the degree of use of vehicles during daily work on the route as the average value of the belonging function, which indicates the degree of compliance with fuzzy restrictions:

$$\overline{\mu_g} = \frac{1}{N-1} \sum_{\omega=z_2}^{z_N} \mu_g^\omega(t), \qquad \overline{\mu_c} = \frac{1}{N-1} \sum_{\omega=z_2}^{z_N} \mu_c^\omega(t) \qquad (6)$$

The unclear objectives of setting the task should be subject to the main criterion for considering and solving the problem of building a traffic schedule and optimal control on the route, this criterion takes into account the cost of working a schedule consisting of the cost of working time of vehicles for servicing passengers and the cost of waiting time for passengers of vehicles on the route:

$$C = c_z \cdot T_z + c_w \cdot T_w,$$

where c_z is the cost of one hour of service of the route by the vehicle and c_w is the cost of passengers for one hour of waiting at stops.

The determining of the optimal time interval t_ω for each time state ω.

If you build a fuzzy model to solve a problem, then the optimal time intervals that will be included in the optimal schedule of work on the route are determined by both the fuzzy target function and the fuzzy restrictions grouped into the system, on a certain temporary state of work on the route:

$$t_\omega = \begin{cases} \arg\max \mu_O^\omega(t) & \text{якщо } \mu_O^\omega(t) \le d, \\ O_{d,n_O}^\omega(t) & \text{якщо } \mu_O^\omega(t) > d, \end{cases} \qquad (7)$$

for $t \in T = \{t_1, \ t_2, \ldots, t_M\}$.

Establishment of a timetable for the operation of vehicles on the passenger service route.

The solution of the set problem will be a matrix O of size $p \times q$, the elements of which will be $O(\omega, t_\omega) = \mu_O^\omega(t)\omega$.

The decision space O is the basis for generating the schedule Z as a service time schedule, in which two consecutive departures of buses by message correspond to states $\omega, \omega + t_\omega$, where is the time interval $- t_\omega$ corresponding to the state ω.

Let's write down the sequential steps for finding the solution space $O(\omega, t_\omega)$:

– First, we assign zero values to all elements of the matrix $O(\omega, t)$, as well $K_j^{\omega,t} = 0$.

– In the cycle by $\omega \in \Omega, t \in T, j = \overline{1, J-1}$, we will calculate the number of passengers who continue to move in the bus after the j-th stop according to the formula given above:

$$K_j^{\omega,t} = K_{j-1}^{\omega,t} + t_\omega \left(v_j^q - \sum_{i=1}^{J-1} v_j^q \cdot \lambda_{ij}^q \right), \text{де } q = \left[\frac{\omega}{\omega_0}\right].$$

– Find the value of belonging functions in the cycle by $\omega \in \Omega, t \in T$:

$$\mu_g^\omega(t) = \sum_{j=1}^{J-1} b_j^q \cdot \mu_{g,B}(K_j^{\omega,t}),$$

$$\mu_c^\omega(t) = \sum_{j=1}^{J-1} b_j^q \cdot \mu_{c,B}(K_j^{\omega,t}),$$

$$\mu_O^\omega(t) = \mu_g^\omega(t) \wedge \mu_c^\omega(t).$$

– In the cycle by $t \in T$, we calculate optimal time intervals, a set corresponding to the solution space in the form of a matrix O:

$$t_\omega = \arg\max \mu_O^\omega(t),$$

$$O(\omega, t_\omega) = \mu_O^\omega(t_\omega).$$

– Place the value of the function $\mu_O^\omega(t)$ by dropping the values of the elements, write the result in a vector column

$$spuskO = descending(\mu_O^\omega(t)).$$

– Perform the following actions in the cycle by $n = \overline{1, n_O}$:

if $spuskO(n) \leq d$ then $t_\omega = spuskO(n),\ \ O(\omega, t_\omega) = spuskO(n)$.

5. Conclusions

In the course of the above algorithm, we will get the schedule of the vehicles on the route in the service interval sequence record. This arrangement will be close to the optimal work schedule, which can be considered given the number of bus flights N, the service time of the route by vehicles T_z, the travel time of passengers on the route T_ω, the waiting time of passengers of the vehicle T_w, the average number of passengers in the bus – P, the average degree of satisfaction of the needs of passengers $\overline{\mu_g}$ and the average value of the degree of use of vehicles during daily work on the route $\overline{\mu_c}$. The cost of rolling stock operation on the route under consideration according to the schedule is calculated as follows:

$$C = c_z \cdot T_z + c_w \cdot T_w$$

References
1. Levkovets, P. R., Moroz, M. M., Kobyletskiy, R. V. (2007). Improving the logistics management of passenger transportation [Udoskonalennia lohistychnoho upravlinnia perevezen pasazhyriv], *Visnyk KDPU imeni Mykhaila Ostrohradskoho,* Vyp. 6 (47), s. 113–115 [in Ukrainian]
2. Ceder, A. (2006). Planning and Evaluation of Passenger Ferry Service in Hong Kong. *Transportation,* Vol. 33, pp. 133–152.
3. Ceder, A., Voß, S., Daduna, J. (2001). Efficient Timetabling and Vehicle Scheduling for Public Transport. Computer-Aided Scheduling of Public Transport. *Lecture Notes in Economics and Mathematical Systems,* Vol. 505, pp. 37–52.

4. Gorbachov, P. F., Lyubiy, Ye. V. (2014). *Modeling of demand for transportation of the population of small towns population by route passenger transport:* monograph [Modeliuvannia popytu na perevezennia naselennia malykh mist marshrutnym pasazhirskym transportom: monohrafiia], KhNADU, Kharkiv, 257 p. [in Ukrainian]
5. Alvarez, A., Casado, S., Gonzalez Velarde, J., Pacheco, J. (2010). A computational tool for optimizing the urban public transport. *Journal of Computer System Sciences International,* Vol. 49 (2), pp. 244–252.
6. Leurent, F. (2011). Transport capacity constraints on the mass transit system: systemic analysis. *European Transport Research Review,* Vol. 3, pp. 11–21.
7. Marieke, S. van der Tuin, Pel, A. J. (2020). The disruption transport model: computing user delays resulting from infrastructure failures for multi-modal passenger & freight traffic. *European Transport Research Review,* Vol. 12, pp. 1–10.
8. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control,* Vol. 8 (3), pp. 338–353.
9. Bellman, R. E., Zadeh, L. A. (1970). Decision-Making in a Fuzzy Environment. *Management Science,* Vol. 17, pp. 141–164.