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METHODOLOGICAL ASPECT
ANALYTICAL FUNCTION OF INDEX METHOD

In term of market economy the necessity of comparing the effectiveness of enterprises performance is becoming increasingly important in order to make definite decisions. The comparison based on particular examples is likely to provide less amount of information than the one based on summary indicators covering a rather wide range of objects, for example, the enterprises of certain industry.

The study of average level changes of this or that phenomenon under the influence of its structural changes is of special interest.

Many economists' and statisticians' works are dedicated to structural shifts of index analysis. However the question of distribution influencing the structural shifts through areas and groups of aggregates is still debatable.

One of the variants of the solution of this question is proposed in the given article.

To prove our further reasoning let us examine the case where the volume of the production output (Q) is its result.

The level of labour productivity is one of the major factors influencing the volume of production output and an enterprise (or in its separate sectors). We will examine the proposed method of distributing through structural shift areas just using this group enterprises indicator.

Before we start our reasoning let us introduce some designations.

Q – the produce volume in its natural value expression (bearing in mind that the same product is produced in different sectors and it can be summed up) or in its monetary means.

T – the total spending of working timetable or average list number of employees (or workers);

$q = \frac{Q}{T}$ – the level of labour productivity;

$dT = \frac{T}{\sum T}$ – the specific position of the sectors (for example, enterprises) in the total labour inputs working timetable

$i_T = \frac{T_1}{T_0}$ –indices T in separate sectors (enterprises);

$I_T = \frac{\sum T_1}{\sum T_0}$ index T in all sectors (enterprises) in the whole.

Let us make one more tentative remark: we will determine the index of average labour productivity of structural shifts with the help of basic balance, i.e. we will use the method of chain substitutions. It should be said that this remark is a specification and in no way the necessary requirement.

$$Y_{\bar{q}\ str.\ shifts} = \frac{\sum q_0 T_1}{\sum T_1} : \frac{\sum q_0 T_0}{\sum T_0} \qquad (1)$$

It is logical for two reasons:

1) the products are the same, expressed either in cost units and it can be directly summed up, so that there is not any necessity to resort to the reciprocal value of product and labour intensiveness aggregates;

2) the initial is the labour inputs structure (T), and not the volume of output (Q).

From the view point of economics and statistics it is quite logical to use the following way of reasoning.

Firstly, though the irregularity of T changes through sectors is the cause of labour inputs structure changes (dT), it does not influence Q by itself, but through the changes $\bar{q}$ (otherwise $I_{\bar{q}\ str.shifts}$ would be meaningless). We are interested not in structural shifts as such, but in their influence on $\bar{q}$ and with the help of the last on Q. From the view point of the economy we will have the output only when labour inputs take place with this or that productivity.

Secondly, with the balance of q_0 in $I_{\bar{q}\ str.shifts}$ it is logical to expect that if the specific position of the sector increases, where $q_0 > \bar{q}_0$, $\bar{q}$ will also increase owing to it. On the contrary, if the specific position of the sector, where $q_0 < \bar{q}_0$, increases, $\bar{q}$

must in its turn decrease due to this. Accordingly, in the first case ΔQ_{dT} will be above zero, in the second case – below zero.

In reasoning in the similar way one can expect that if the specific position of the sector decreases, where $q_0 > \overline{q}_0$, $\overline{q}$ will decrease and ΔQ_{dT} in this sector will be below zero, if, on the contrary, the specific position of the sector decreases, where $q_0 < \overline{q}_0$, the $\overline{q}$ will increase and ΔQ_{dT} will be signed with "plus".

These logically, economically and statistically contradictory conclusions may be represented in schematic form.

Table 1

The influence of labour inputs, structure changes on the changes of average labour productivity and the volume of product output

Correlation of q_0 and $\overline{q}_0$ and the sign $q_0 - \overline{q}_0$	Change of dT and the sign $dT_1 - dT_0$	Expected change $\overline{q}$ and Q at the cost of dT (sign ΔQ_{dT})
$q_0 - \overline{q}_0 > 0\,(+)$	$dT_1 - dT_0 > 0\,(+)$	$\Delta Q_{dT} > 0\,(+)$
$q_0 - \overline{q}_0 < 0\,(-)$	$dT_1 - dT_0 > 0\,(+)$	$\Delta Q_{dT} < 0\,(-)$
$q_0 - \overline{q}_0 > 0\,(+)$	$dT_1 - dT_0 < 0\,(-)$	$\Delta Q_{dT} < 0\,(-)$
$q_0 - \overline{q}_0 < 0\,(-)$	$dT_1 - dT_0 < 0\,(-)$	$\Delta Q_{dT} > 0\,(+)$

This scheme can be supplemented by particular cases, when either 1) $q_0 = \overline{q}_0$, or 2) $dT_1 = dT_0$. In the first case the change of dT must in no way affect $\overline{q}$, and ΔQ_{dT} within this sector will equal zero. With $dT_1 = dT_0$ within this sector we will have $\Delta Q_{dT} = 0$, as the specific weight would remain unchangeable.

The mathematical "rules of signs" is used as to multiplication of positive and negative numbers both in more common cases in the table and in specific cases.

One can say that this similarity with mathematics has a formal character. However, we did not start with this similarity, but with economic and statistics reasonings. The similarity here may play the role of "mnemonic rule" ("the rule of signs"). But the role of similarity is supposed to be not only in this case.

To illustrate our further reasoning let us examine the provisional example of an aggregate, consisting of 5 enterprises.

Table 2

For calculating structural shift index

Enterprise №	Labour productivity		Labour costs for production output		The volume of produced products	
	q_0	q_1	T_0	T_1	Q_0	Q_1
1	16	16	10	10	160	160
2	20	20	30	30	600	600
3	24	24	60	66	1440	1584
4	25	25	40	66	1000	1650
5	30	30	60	48	1800	1440
$\sum$ *if* $\overline{X}$	25	24,7	200	220	5000	5434

The calculation of structural shift index according to initial formula gives the following result:

$$I_{str.\,shifts} = \frac{\sum q_0 T_1}{\sum T_1} : \frac{\sum q_0 T_0}{\sum T_0} = \frac{5434}{220} : 25 = 24{,}7 : 25 = 0{,}988.$$

The literature offers different approaches to the solution of the problem of distributing the influence of structural shifts within the sectors and groups of aggregate. However, their realization not always leads to substantiated and logically interpreted economic conclusions.

We would like to offer you some nonstandard ways of solving this question. Mathematical statistics proves that some indices, including the index of structural shifts, can be expressed with the help of correlation ratio and variation coefficient. Here is the expression in its general view:

$$I_{str.\,shifts} = \frac{\sum x_0 Y_1}{\sum Y_1} : \frac{\sum x_0 Y_0}{\sum Y_0} = 1 + r_{x_0 i_Y} \nu_{Y_0} \nu_{i_Y}, \qquad (2)$$

where x is the quality indicator,

y is the volume indicator which admits summing up,

r is the correlation coefficient,

ν is the variation coefficient.

In these calculations r and ν media are not simple, but are weighed in Y_0.

Applying to the example in question $x = q$, $Y = T$, $xY = Q$.

It gives:

$$I_{\substack{\bar{q}\\ str.\,shifts}} = \frac{\sum q_0 T_1}{\sum T_1} : \frac{\sum q_0 T_0}{\sum T_0} = 1 + r_{q_0 i_T} \nu_{q_0} \nu_{i_T}, \qquad (3)$$

where i_T is an individual index T.

With averaging q_0 and i_T T_0 is the scale.

And if we proceed from the formula 3, where mathematical statistics indicators are present, to the common formula in which only index indicators and denominations are used, and preserving the right hand side pattern, we will get:

- correlation coefficient:

$$r_{q_0 i_T} = \frac{\sum (q_0 - \bar{q}_0)(i_T - I_T) T_0}{\sum T_0 \sigma_{q_0} \sigma_{i_T}},$$

- variation coefficient:

$$\nu_{q_0} = \frac{\sigma_{q_0}}{\bar{q}_0}, \qquad \nu_{i_T} = \frac{\sigma_{i_T}}{I_T}.$$

Therefore, the second constituent in the right hand side pattern of the formula (3) – $r_{q_0 i_T} \nu_{q_0} \nu_{i_T}$ – after evident transformations (cancellation by σ_{q_0} and σ_{i_T}) we will have the following figuration:

$$r_{q_0 i_T} \nu_{q_0} \nu_{i_T} = \frac{\sum (q_0 - \bar{q}_0)(i_T - I_T) T_0}{\bar{q}_0 I_T \sum T_0} = \frac{\sum (q_0 - \bar{q}_0)(i_T - I_T) T_0}{\bar{q}_0 \sum T_1} \qquad (4)$$

As a result we will have:

$$I_{\substack{\bar{q}\\ str.\,shifts}} = 1 + \frac{\sum (q_0 - \bar{q}_0)(i_T - I_T) T_0}{\bar{q}_0 \sum T_1}. \qquad (5)$$

It can be shown that after reducing to common denomination and making some changes the formula (5) is taken to its initial view. This acknowledges the validity of the undertaken transformations of this index.

But from the viewpoint of distributing within the sectors the changes of product volume at the account of labour cost structural shifts (ΔQ_{dT}) the formula (5) is of special interest.

Let us calculate the structural shift index.

Table 3

For calculating the structural shift index

Enterprise №	q_0	T_0	T_1	i_T	$q_0 - \bar{q}_0$	$i_T - I_t$	$(q_0 - \bar{q}_0)(i_T - I_T)T_0$
1	16	10	10	1,00	- 9	- 0,10	9
2	20	30	30	1,00	- 5	- 0,10	15
3	24	60	66	1,10	- 1	0	0
4	25	40	66	1,65	0	0,55	0
5	30	60	48	0,80	5	- 0,30	- 90
$\sum$ *if* $\bar{X}$	25	200	220	1,10	-	-	- 66

$$I_{\bar{q}\ str.shifts} = 1 + \frac{-66}{25 \cdot 220} = 1 - 0{,}012 = 0{,}988,$$

i.e. is the same as according to initial formula.

In the last column of table (3) are the changes of product volume at the account of labour cost structural shifts (ΔQ_{dT}), that completely accord with the above stated reasoning about the influence of structural shift changes (dT) on $\bar{q}$ and on Q, as well as it accords with “the rule of signs”.

Thus, for enterprise 1 and enterprise 2 the specific weight of each decreased and $q_0 < \bar{q}_0$, i.e. $q_0 - \bar{q}_0$ has the sign “minus”, so we have a positive contribution in total amount of increase in production at the account of dT. The specific weight of enterprise 3 has not changed, its contribution equals zero. At the enterprise $q_0 = \bar{q}_0$, so despite a sharp increase of its specific weight (from 20 % to 30 %), it did not influence это $\bar{q}$ and $\Delta Q_{dT} = 0$. And only for enterprise 5, where $q_0 > \bar{q}_0$ and whose specific weight decreased, we have $\Delta Q_{dT} < 0$, that exceeded the positive accession for enterprise 1 and 2 and it stipulated the fact that $I_{\bar{q}\ str.shifts} < 1$, and the total accession is $\Delta Q_{dT} < 0$.

The given methodic can be applied not only for a particular enterprise or industry, but for the economy as a whole. It is self understood that while analyzing the regional and country’s economy as a whole it is lawful to speak about and increase or decrease of production only if their structure remains unchanged. But when the structure in this or that respect is changing (sectoral, territorial and other shifts) it is necessary to take it into account.

The proposed methodic in principal may be used in all cases, when the unification separate units of an aggregate gives some positive effect (aggregate effect). Its distribution among separate units (sectors, enterprises etc.) gives the possibility to determine its contribution in this additional effect.