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INVESTMENT DECISION BASED ON ANALYSIS OF MATHEMATICAL INTERRELATION BETWEEN CRITERIA IRR, MIRR, PI

ABSTRACT

Analysis of the advantages and disadvantages of indicators of the internal rate of return and the modified internal rate of return as criteria for the effectiveness of an investment project. The mathematical and graphic apparatus for studying the functions and dependencies between the economic characteristics of future financial and production activities of entrepreneurial activity was used in the process of the analysis of the most important indicators of the effectiveness of the investment project. The characteristics of the criterion of the internal rate of return are studied on the basis of the properties of the function which describes the dependence of the net present value of the project on the value of the discount rate. The main advantages of the modified internal rate of return in comparison with its unmodified analogue are revealed. The inequalities between the indicated indicators are mathematically proved, their influence on the stability (safety) characteristics of the investment project is analyzed, and the interrelation of the criterion of the modified internal rate of return with the value of the profitability index is shown. The scientific novelty consists in the mathematical proof of the interrelation between the indicators of the internal rate of return and the modified internal rate of return, as well as the interrelation between the criterion of the modified internal rate of return and the profitability index of the investment project. Theoretical conclusions and suggestions can be used in the investment analysis of future financial and production projects in the domestic economy, which opens up the possibilities of rational use of resources in entrepreneurial activity at all levels of business process management.

Keywords: investment project, profitability index, net present value, non-ordinary cash flow, investment

JEL Classification: E22, F 37, F 21, G31

INTRODUCTION

The current economic crisis, one of the main factors of which is the COVID-19 pandemic, has affected all, without exception, developed and developing countries of the world. The global economy decreased by 4.3%, which is more than two and a half times more than during the 2009 global financial crisis, in 2020. According to the released UN World Economic Report, the expected moderate recovery in 2021 will hardly compensate the losses of the previous year.

Developed countries, for which growth of production by about 4% in 2021 is predicted, have been hit hard during the pandemic. Their economies decreased by an average of 5.6% in 2020. The drop was mainly due to forced shutdowns in manufacturing and services, as well as due to premature austerity measures that could only further complicate the situation (Lishchynskyy, I. et al, 2020) [9]. As for developing countries, according to estimates presented in the UN World Economic Report, there was a less serious decrease in the rate of economic development (2.5%) with an expected recovery of 5.7% in 2021.

Experts of the World Economic Forum warned that the consequences of the coronavirus pandemic could threaten the development of the global economy in the next 3-5 years. In addition, as a result of the pandemic, social inequality may increase and geopolitical stability may weaken over the next 5-10 years (Global Risks Report, 2021) [20].

Top managers of companies should pay special attention to the rational use of all available resources and, in particular, those that are directed to investing in new products and financial projects in these economic conditions (Durán et al, 2018) [3]. Indicators and criteria of investment analysis, taking into account the change in the value of money over time by discounting and increasing are serving this purpose.

In the scientific literature on investment analysis, theoretical discussions about the advantages and disadvantages of these indicators and criteria, which have serious practical importance for business representatives in all countries of the world, still do not subside (Lyzun et al, 2019) [10]. Therefore, in this article, we tried to highlight the disadvantages of the Internal Rate of Return indicator as a characteristic of the effectiveness of the studied project and demonstrate some important mathematical interrelations between the investment analysis criteria.

LITERATURE REVIEW

In his paper Arjunan (2017) evaluates whether MIRR is an appropriate criterion for investment decisions and the true annual rate of return on capital [1]. The estimation of MIRR is based on the modified net cash flow (MNCF). The MNCF, derived by mathematically adjusting the actual net cash flow (NCF), distorts the intrinsic value of the cash inflow and its timing. With MNCF, the MIRR is lower than the IRR because MIRR failed to fully utilize the NCF generated as shown by the capital amortization schedule. The estimated MIRR, based on the assumed reinvestment rate, leads to serious problems as explained above. MIRR (when MIRR < IRR) estimate does not fully utilize the benefit stream. Based on these results, it is evident that the MIRR is a spurious criterion.

Kukhta (2014) writes that the MIRR method is more attractive than the IRR as a characteristic of the real profitability of the project (or the expected long-term rate of return of the project), but NPV (Net Present Value) is still better for analysing alternative projects that differ in scale because it shows in absolute terms, how much the optimal project increases the value of the company [6]. A method of MIRR is indispensable for the evaluation of atypical projects, where the usual IRR shows erroneous or ambiguous results. It is hoped that the method of modified internal rate of return will obtain the same popularity as its predecessor, the original IRR (Tran et al, 2020) [21]. Due to its properties, this method also ensures the confidentiality of project information, in contrast to NPV, which to some extent clarifies the scale of the project. It can be used as the main criterion when approving materials for loans for international projects, as it eliminates the necessity to compare discount rates in different countries or calculate the "global" discount rate (Kuznetsova, A. Y. et al, 2019) [7].

Yankovyi and Melnyk (2018) [24] point to the threat of using the Internal Rate of Return indicator because of its potential significant overestimation of the efficiency of an investment project in some cases. They recommend paying more attention to the Modified Internal Rate of Return criterion as a universal indicator of the relative profitability of a planned project.

Mytskikh (2019) [13] casts doubt on the advisability of using the MIRR criterion as an indicator of the effectiveness of an investment project. In the conclusions of her article, she, in particular, asserts that the MIRR indicator really allows you to rank projects consistently with the ranking by the NPV indicator, but only for alternative projects of the same scale. The MIRR criterion of the original project is in fact the IRR criterion of the substitution project, which is not equivalent to the original project. MIRR has many values because it is a function of the increasing rate (capital price). A set of MIRR values can also take place at a given capital price, but at different periods of increasing (reinvestment). The internal rate of return of the project is a characteristic of the project, and it should not depend on the price of capital used in the project, therefore, the MIRR indicator cannot act as an indicator of the internal rate of return of the original project. In general, the MIRR indicator cannot be used in the formation of the capital investment budget.

Hayes (2021) [4] writes that cash flows are often reinvested at the cost of capital, not at the same rate at which they were generated in the first place. IRR assumes that the growth rate remains constant from project to project. It is very easy to overstate the potential future value with basic IRR figures. Another major issue with IRR occurs when a project has different periods of positive and negative cash flows. In these cases, the IRR produces more than one number, causing uncertainty and confusion. The modified internal rate of return improves on the standard internal rate of return value by adjusting differences in the assumed reinvestment rates of initial cash outlays and subsequent cash inflows.

According to Ross (2021) [15], the formula for modified internal rate of return allows analysts to change the assumed rate of reinvested growth from stage to stage in a project. The most common method is to input the average estimated cost of capital, but there is flexibility to add any specific anticipated reinvestment rate. The MIRR also is designed to generate one solution, getting rid of the problem of multiple IRRs.

Thus, the analysis of literary sources allows us to conclude that the opinions of scientists about the expediency of applying the IRR and MIRR criteria in the process of analysing the economic efficiency of investment projects were divided. The

first group of researchers (K. Arjunan, N. Mytskikh) believes that the MIRR indicator is a false criterion since its comparison with the discount rate can lead to incorrect (underestimated) conclusions regarding the degree of a project's efficiency. The second group of scientists (P. Kukhta, O. Yankovyi and N. Melnyk, A. Hayes, S. Ross) is of the opposite opinion. They argue that the MIRR criterion improves the standard internal rate of return and is designed to generate a single solution that helps to get rid of the problem of multiple IRRs typical of non-ordinary cash flow.

AIMS AND OBJECTIVES

The research objective is to prove mathematically the interrelation between the indicators of the internal rate of return and the modified internal rate of return, as well as the interrelation between the criterion of the modified internal rate of return and the profitability index of the investment project.

METHODS

In the process of assessing the effectiveness of the company's investment projects, five main criteria are identified, which are based on changes in the value of money over time: 1. Net Present Value (NPV); 2. Profitability Index (PI); 3. Internal Rate of Return (IRR); 4. Modified Internal Rate of Return (MIRR); 5. Discounted Payback Period (DPP).

Let us introduce the following designations:

PV – discounted current total value of cash receipts from the project;

$$PV = \sum_{k=1}^{n} \frac{P_k}{(1+r)^k}$$

where Pk – receipts from the project in the kh year (k = 0, 1, 2, …, n); IC – discounted current total value of an investment in the project;

$$IC = \sum_{k=0}^{n} \frac{IC_k}{(1+r)^k}$$

where ICk – investment in the project in the kh year; n – duration of the project (years); r – discount rate, which is based on a risk-free rate, risk premium, percentage of inflation; Norm – a normative value of payback, or reserve of security of the project.

Then the formulas of these five criteria with conclusions on the acceptability of the investment project are presented in table 1.

Table 1. Discounted economic criteria and indicators of investment projects.

Indicator	Calculation formula	Conclusion on project acceptability
1. *Net Present Value* (*NPV*)	$NPV = PV - IC$	*NPV* > 0 – the project is acceptable *NPV* < 0 – the project is not acceptable *NPV* = 0 – decision is not defined
2. *Profitability Index* (*PI*)	$PI = \dfrac{PV}{IC}$	*PI* > 1 – the project is acceptable *PI* < 1 – the project is not acceptable *PI* = 1 – the decision is not defined
3. *Internal Rate of Return* (*IRR*)	$IRR \approx r_1 + \dfrac{f(r_1)(r_2 - r_1)}{f(r_1) - f(r_2)}$	*IRR* > *r* – the project is acceptable *IRR* < *r* – the project is not acceptable *IRR* = *r* – the decision is not defined
4. *Modified Internal Rate of Return* (*MIRR*)	$MIRR = \sqrt[n]{\dfrac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{\sum_{k=0}^{n} \frac{IC_k}{(1+r)^k}}} - 1$	*MIRR* > *r* – the project is acceptable *MIRR* < *r* – the project is not acceptable *MIRR* = *r* – the decision is not defined
5. *Discounted Payback Period* (*DPP*)	DPP = min n, when $PV \geq IC$	*DPP* < *Norm* – the project is acceptable *DPP* > *Norm* – the project is not acceptable *DPP* = *Norm* – the decision is not defined

During the investment analysis, the top managers of the company themselves determine the range of indicators-criteria from table 1, which will assess future production and financial measures. Therefore, the specific qualitative and quantitative properties and characteristics of each of these economic indicators of the investment project are crucial to obtain correct conclusions about the acceptability of individual proposed projects, as well as for the investment portfolio of the company in general.

RESULTS

Characteristics of the Internal Rate of Return criterion. By definition, IRR is the discount rate of all positive and negative cash flows of an investment project when their amounts are equal to each other, i.e. NPV = 0. (Figure 1).
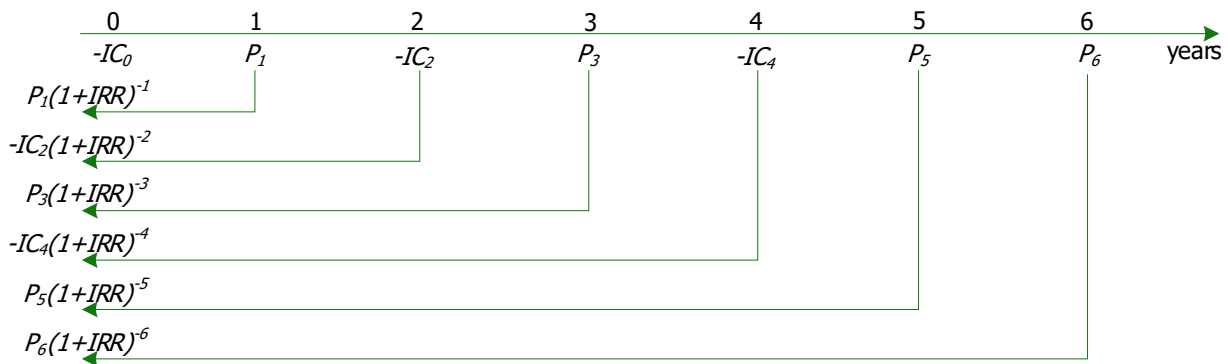

Figure. 1. Scheme of calculation of IRR criterion for an investment project with non-ordinary cash flow.

Therefore, Internal Rate of Return is the breakeven point of the investment project based on the IRR discount rate of cash flows. It characterizes the maximum acceptable relative level of costs possible during the implementation of this project without losses for the owner. Therefore, it is a comparison base when determining the stability (safety reserve) of the project to a change in the discount rate r. Obviously, the project should be assessed as acceptable if IRR > r, unacceptable if IRR < r, and undefined if IRR = r.

According to the definition of Internal Rate of Return, the following equation can be written:

$$\sum_{k=1}^{n} \frac{P_k}{(1+IRR)^k} = \sum_{k=0}^{n} \frac{IC_k}{(1+IRR)^k} \tag{1}$$

Here IC0 means primary investment. Therefore, (1) can be represented as follows:

$$\sum_{k=1}^{n} \frac{P_k}{(1+IRR)^k} = \sum_{k=1}^{n} \frac{IC_k}{(1+IRR)^k} + IC_0 \tag{2}$$

Formulas (1), (2) are equations of the n degree relatively to IRR, therefore calculation of its value directly in the general case is impossible. The interpolation method is usually used by sequential changing the discount rate r, which brings the NPV value closer to zero with a given accuracy to determine the IRR value (see the calculation formula in the 3rd row of Table 1).

To properly understand the nature of Internal Rate of Return, the graphical method of analysis of the function NPV = f(r) is used, which according to the definition of Net Present Value has the following form:

$$NPV = f(r) = \sum_{k=1}^{n} \frac{P_k}{(1+r)^k} - \sum_{k=0}^{n} \frac{IC_k}{(1+r)^k} = \sum_{k=0}^{n} \frac{PN_k}{(1+r)^k} \tag{3}$$

Let us consider the more important properties of function (3):

the function NPV = f (r) is a nonlinear function of r;

when r = 0 f(r) =$\sum_{k=0}^{n} PN_k$, i.e. the graph of the function crosses the Y-axis at a point equal to the sum of all elements of the undiscounted cash flow generated by this investment project (Figure 1);

for projects with ordinary cash flow and with positive NPV (so-called classical cash flow) function (3) is not increasing, i.e. at $r \to +\infty$ its graph crosses the abscissa at some point, which is the IRR according to its definition (Figure 2);

for projects with non-ordinary cash flow and with positive NPV, function (3) due to its nonlinearity can have several real roots, and its graph can cross the abscissa axis at several points (Figure 3).

due to the fact that function (3) is nonlinear, the IRR criterion is a relative and non-additive indicator. It, like the discount rate r, is a decimal fraction.
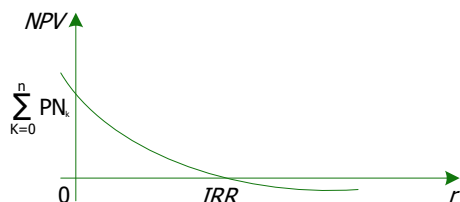

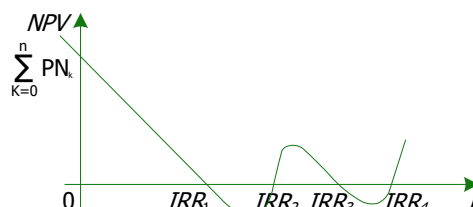**Figure. 2. NPV=f(r) function graph of the with ordinary cash flow flow.**

Figure. 3. NPV=f(r) function graph of the project with non-ordinary cash.

if we change the signs to the opposite in the scheme of cash flow generated by investment project A, in all inflows and outflows, i.e. we obtain cash flow of the new project –A, which is symmetrical about the time axis, the graph of its functions NPV = f(r) will be symmetrical about the axis r to the initial cash flow of the project A.

for two closed in NPV value alternative projects C and D we can observe the so-called Fisher point on the graph of their functions NPV = f(r) (Figure 4).
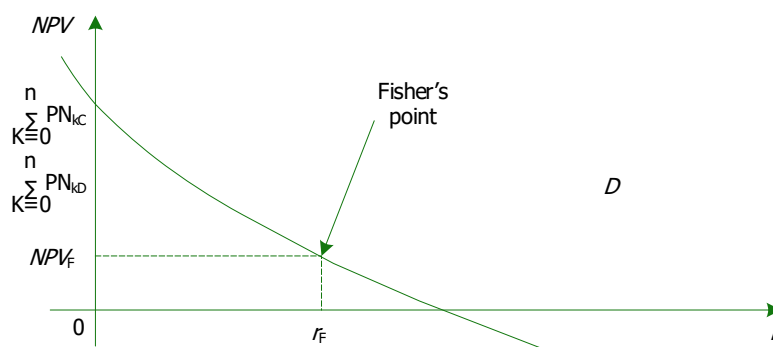

Figure. 4. Graphs of NPV = f(r) functions and Fisher's point with coordinates (NPVF, rF) for two investment projects C and D.

Investment projects change their priority according to the NPV indicator in the Fisher's point, which is the main property of this point. Indeed, to the left of the point rF in Figure 4 NPV(C) > NPV(D), and to the right NPV(C) < NPV(D).

Fisher's point can be found when equation (4) has at least one root. Here PNk, PNl are the corresponding cash flows of projects C and D, the duration of which is n and g years.

$$\sum_{k=0}^{n} \frac{PN_k}{(1+r_F)^k} = \sum_{l=0}^{g} \frac{PN_l}{(1+r_F)^l} \tag{4}$$

From expression (4) follows the interrelation

$$\sum_{k=0}^{n} \frac{PN_k}{(1+r_F)^k} - \sum_{l=0}^{g} \frac{PN_l}{(1+r_F)^l} = 0 \tag{5}$$

This means that the value of rF in Figure 4 can be considered as the internal rate of return of a conditional investment project (C - D), the cash flow of which is equal to the difference between the corresponding inflows and outflows of projects C and D, and its duration - the value of max (n; g). According to property 6 of the function NPV = f(r) for the conditional project (D - C) the value of rF does not change.

Obviously, in the case of the non-ordinary cash flow of a new investment project (C - D) or (D - C), equation (5) may have not one but several roots. This allows us to make a conclusion about the fundamental possibility of finding several of Fisher's points. In addition, equation (5) may not have real roots at all and Fisher's point is not observed.

The disadvantages of the IRR criterion are usually attributed by some researchers to the lack of properties of choosing the only best investment project from many possible (alternative, interdependent, etc.) (Juma, Kilani, 2022) [5]. Although it has already been proven long ago that the correct decision about the acceptance or rejection of future investment is achieved only with the integrated use of all indicators in Table 1. For example, it is often noted that there is a contradiction between the NPV and IRR indicators of projects that differ significantly in scale or in terms of implementation (Magni, Marchioni, 2020) [11].

However, there are two disadvantages of the IRR criterion, which are immanently inherent in this criterion and are admitted by the overwhelming majority of specialists in the field of investment analysis:

1. According to formula (1), which is the basis for calculating IRR, all additional investment in the project is discounted by the Internal Rate of Return. Although it would be correct to discount reinvested funds in the project at a rate of r. If the values of IRR and r are not very different from each other, the distortion of the efficiency of a project is insignificant (Das, 2019) [2]. However, when IRR is much higher than r, the Internal Rate of Return is subject to significant and unjustified overstatement. In this regard, Kelleher and MacCormack (2004) [12] wrote, "Managers of one large industrial company in five years have approved 23 large capital projects based on IRR indicators, which averaged 77%. However, when we recently conducted an analysis, equating the reinvestment rate to the value of the company's capital, the average income fell to 16%. The most interesting thing is that when we recalculated the IRR of the three projects, which were considered the most profitable, using a realistic reinvestment rate, we found that they had fallen from 800, 150 and 130% to 15, 23 and 22%, respectively. Unfortunately, decisions to invest in those projects have already been made. Of course, such high IRR rates are atypical. But even if the IRR of the project is reduced from 25 to 15%, it is very significant».

2. Property of the function NPV = f (r) for investment projects with non-ordinary cash flows means that equation (1) can have several real roots, and the graph of the function can intersect the abscissa axis at several points (Figure 1). In this case, there is uncertainty due to the plurality of IRR values. For example, in Figure 2 the positive value of Net Present Value is observed in segments [0; IRR1], [IRR2; IRR3], and [IRR4; +∞]. In addition, for investment projects with non-ordinary cash flows, equation (1) may have no real roots at all, and the graph of the function NPV = f (r) may not intersect the abscissa axis (see project D in Figure 4). In this situation, the value of the Internal Rate of Return is not possible to determine either.

Characteristics of the Modified Internal Rate of Return criterion and its comparison with other indicators of investment analysis. MIRR is a discount rate at which the terminal value of the project is reduced to the present moment and is equal to the present value of all investments associated with this project. MIRR, by definition, is a modification of the Internal Rate of Return indicator, which is designed to eliminate the two above-mentioned disadvantages of the IRR criterion (Rebiasz, 2020) [14]. The MIRR and IRR indicators for this investment project are fully consistent, i.e. if IRR > r, the inequality MIRR > r is satisfied, if IRR < r, then MIRR < r and with IRR = r, the equality MIRR = r is solved with the corresponding conclusions on the acceptance (rejection) of the project under study.

Like IRR, the Modified Internal Rate of Return is a relative financial measure of the efficiency of an investment project, which is often used when budgeting capital investment of companies in order to rank alternative investments of approximately the same size.

By definition, MIRR is a discount rate at which the equality of two values is achieved (Yankovyy, Melnyk, 2021) [23]:

all investment costs are reduced to a discount rate r at the beginning of the project;

cash receipts accrued at the end of the project - the net terminal value of the NTV project, which is calculated according to the following formula:

$$NTV = \sum_{k=1}^{n} P_k(1+r)^{n-k} \qquad\qquad (6)$$

As these values refer to different moments of the project implementation, they should be made commensurate (brought to the beginning of the project) with the help of some discount rate, which is called the modified internal rate of return MIRR (Figure 5).
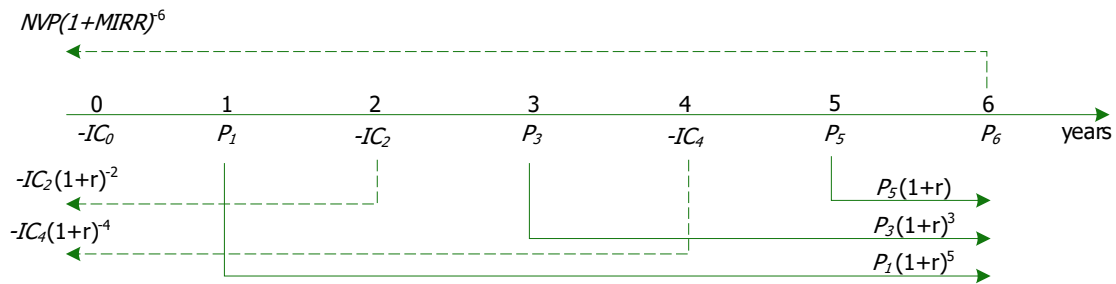

Figure. 5. Scheme for calculating the MIRR criterion (dashed lines show reducing all investment costs and revenues to those existing at the beginning of the project).

Calculations of the MIRR value are based on the following equation, which follows from the conditions of its determination

$$\frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{(1+MIRR)^n} = \sum_{k=0}^{n} \frac{IC_k}{(1+r)^k} \tag{7}$$

It always provides a single solution, unlike equation (1), which determines multiplicity of value IRR for a project with non-ordinary cash flow.

Equation (7) provides the final formula for calculating the needed Modified Internal Rate of Return:

$$MIRR = \sqrt[n]{\frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{\sum_{k=0}^{n} \frac{IC_k}{(1+r)^k}}} - 1 \tag{8}$$

Therefore, the application of the same discount rate r, which is calculated by the top management of a company itself on the basis of a risk-free rate, risk premium, inflation rate, etc., shows that the MIRR criterion (8) manages to eliminate both of the above-mentioned limitations of the Internal Rate of Return indicator.

Establishing a relationship between the studied criteria. By analogy with the IRR indicator (formula (2)), equation (7) can be represented as follows:

$$\frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{(1+MIRR)^n} = \sum_{k=1}^{n} \frac{IC_k}{(1+r)^k} + IC_0 \tag{9}$$

Let us now consider the interrelation between the IRR and MIRR values for the same investment project with non-ordinary cash flows. Comparison of expressions (2) and (9) allows us to write:

$$\sum_{k=1}^{n} \frac{P_k}{(1+IRR)^k} - \sum_{k=1}^{n} \frac{IC_k}{(1+IRR)^k} = \frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{(1+MIRR)^n} - \sum_{k=1}^{n} \frac{IC_k}{(1+r)^k} \tag{10}$$

Using elementary transformations, equation (10) is reduced to the following form:

$$\frac{\sum_{k=1}^{n} P_k(1+IRR)^{n-k}}{(1+IRR)^n} = \frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{(1+MIRR)^n} + \left[\sum_{k=1}^{n} \frac{IC_k}{(1+IRR)^k} - \sum_{k=1}^{n} \frac{IC_k}{(1+r)^k}\right] \tag{11}$$

Let us analyse three possible situations concerning the acceptance (rejection) of the investment project under study.

1. Uncertainty of conclusions: r = IRR. In this case, the interrelation (11) becomes the equality

$$\frac{\sum_{k=1}^{n} P_k(1+IRR)^{n-k}}{(1+IRR)^n} = \frac{\sum_{k=1}^{n} P_k(1+IRR)^{n-k}}{(1+MIRR)^n} \tag{12}$$

which results in IRR = MIRR. Therefore, in this situation, all three criteria are equal to one another (r = IRR = MIRR), as shown in Figure 6.
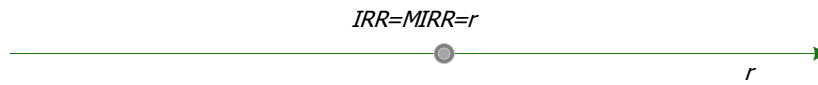
$$IRR=MIRR=r$$

Figure. 6. Relationship between the IRR, MIRR criteria and a discount rate r for an investment project under conditions of uncertainty.

2. An investment project is accepted: r < IRR. In this case, in interrelation (11), the difference in square brackets is less than zero, since $\sum_{k=1}^{n} \frac{IC_k}{(1+IRR)^k} < \sum_{k=1}^{n} \frac{IC_k}{(1+r)^k}$. And the equality in (11) is possible on one condition:

$$\frac{\sum_{k=1}^{n} P_k(1+IRR)^{n-k}}{(1+IRR)^n} < \frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{(1+MIRR)^n} \tag{13}$$

The specified condition is fulfilled only when $(1 + IRR)^n > (1 + MIRR)^n$. Consequently, when accepting an investment project, the following inequality is valid: MIRR < IRR (Figure 7).

Figure. 7. Relationship between the IRR, MIRR criteria and a discount rate r for an investment project, if it is accepted.

3. An investment project is rejected: r > IRR. In this case, in interrelation (11), the difference in square brackets is a positive value, since $\sum_{k=1}^{n} \frac{IC_k}{(1+IRR)^k} > \sum_{k=1}^{n} \frac{IC_k}{(1+r)^k}$. The equality in (11) is possible on such a condition:

$$\frac{\sum_{k=1}^{n} P_k(1+IRR)^{n-k}}{(1+IRR)^n} > \frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{(1+MIRR)^n} \tag{14}$$

It can be satisfied when $(1 + IRR)^n < (1 + MIRR)^n$. Consequently, if an investment project is rejected, the following relationship takes place: MIRR > IRR (Figure 8).

Figure. 8. Relationship between the IRR, MIRR criteria and a discount rate r for an investment project, if it is rejected.

It is obvious that in the case of ordinary cash flows, all those relationships between the IRR and MIRR criteria for the same investment project, which have been proven above and presented in Figures 6, 7, 8 are also observed. Admittedly, in this situation, the term on the right-hand side of (11) is equal to zero and all the arguments given for the case of non-ordinary cash flows are valid.

The IRR and MIRR criteria as indicators of the effectiveness of an investment project can be used to find the absolute (relative) Margin of Strength (MS), which characterises the sustainability and safety of the planned event:

$$MS_{IRR} = IRR - r; \qquad MS'_{IRR} = \frac{(IRR-r)}{r} \times 100$$

$$MS_{MIRR} = MIRR - r; \quad MS'_{MIRR} = \frac{(MIRR-r)}{r} \times 100 \tag{15}$$

The absolute (relative) MS of a project reflects the possible margin of variation of the actual values of the IRR, MIRR of a project, caused by various unforeseen circumstances, such as an increase in the discount rate r. In particular, if MS > 0, expressions (15) show by how many units (per cent) the potential profitability of an investment project can decrease without drastically changing its acceptability, i.e. without turning the project from profitable to unprofitable.

Evidently, the higher the MS indicator is, the greater the margin of stability (safety) of the planned event is, the lower its riskiness becomes and vice versa. If MS <0, then its value indicates by how many units (per cent) it is necessary to increase the profitability of an investment project in order to turn it from unprofitable to profitable.

Figure 7 clearly shows that if a project is accepted, the calculation according to formulas (15) ensures the solution to the inequalities $MS_{IRR} > MSM_{IRR} > 0$, $MS'_{IRR} > MS'_{MIRR} > 0$. This means that in this situation the Internal Rate of Return criterion gives a significantly overstated estimate of the stability (safety) of an investment project in comparison with the Modified Internal Rate of Return indicator.

Figure 8 illustrates that if a project is rejected, the calculation by formulas (15) proves the validity of the inequalities $MS_{IRR} < MS_{MIRR} < 0$, $MS'_{IRR} < MS'_{MIRR} < 0$. Therefore, in this case, in comparison with the Modified Internal Rate of Return indicator, the Internal Rate of Return criterion also gives an overstated estimate – but not of the stability (safety) of an investment project; the criterion overestimates the necessary growth in a project's profitability and transformation from unprofitable to profitable.

In our opinion, it is useful to compare the MIRR indicator not only with the internal rate of return IRR, but also with the Profitability Index (PI) as it is another characteristic of the efficiency of an investment project. Let us consider carefully the expressions that define them:

$$PI = \frac{PV}{IC} = \frac{\sum_{k=1}^{n} P_k(1+r)^{-k}}{\sum_{k=0}^{n} IC_k(1+r)^{-k}}. \tag{16}$$

In formula (16), the discount rate of cash inflows and outflows of a project is presented as a factor $(1 + r)^{-k}$. Based on expression (7) of the MIRR criterion, we can get the following relationship:

$$(1 + MIRR)^n = \frac{\sum_{k=1}^{n} P_k(1+r)^{n-k}}{\sum_{k=0}^{n} IC_k(1+r)^{-k}}. \tag{17}$$

A comparison between formulas (16) and (17) shows that they differ only in the factor $(1 + r)^n$. Therefore, the following expressions are valid:

$$PI = \left(\frac{1+MIRR}{1+r}\right)^n; \qquad MIRR = (1+r)PI^{\frac{1}{n}} - 1 \tag{18}$$

Formulas (18) mean that the MIRR and PI indicators are interrelated and can be expressed through each other. Moreover, their values are completely consistent with each other. Indeed, according to the first relationship (18):

- when r = MIRR (Figure 6) PI = 1 the situation regarding the acceptance of an investment project is uncertain;
- when r < MIRR (Figure 7) PI > 1 - a project is acceptable;
- when r > MIRR (Figure 8) PI < 1 - a project is unacceptable.

DISCUSSION

Under the conditions of global economic recession, the question of increasing the efficiency of the economic activity of enterprises and companies is particularly acute. Thus, great attention should be paid to the rational use of all available resources and, in particular, those directed to investment in new products and financial projects. To achieve this goal, it is advisable to use modelling of such economic processes based on indicators and criteria of investment analysis, which take into account the change in the value of money over time by discounting and increasing. This paper identifies the bottlenecks and shortcomings of the Internal Rate of Return indicator as performance characteristics for the researched project and demonstrates some important mathematical relationships between investment analysis criteria. Mathematically proven relationships between indicators of the Internal Rate of Return and the Modified Internal Rate of Return facilitate the investment analysis of future financial and production projects in the economy and thus contribute to the rational use of resources in business activities at all levels of business process management.

CONCLUSIONS

The mathematical results of the study prove that on the condition that an investment project is adopted, the Internal Rate of Return criterion overestimates its effectiveness and the degree of its stability (safety) in comparison with the Modified

Internal Rate of Return indicator. When a project is rejected, the Internal Rate of Return criterion also gives an overstated estimate, but not of the stability (safety) of an investment project as the Modified Internal Rate of Return indicator does; the criterion overestimates the necessary increase in profitability and turning this project from unprofitable to profitable.

In addition, it has been shown that such criteria as the Modified Internal Rate of Return and the Profitability Index are interconnected and can be expressed through each other. Moreover, in the process of testing an investment project, their values are fully consistent with each other

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ІНВЕСТИЦІЙНЕ РІШЕННЯ НА ОСНОВІ АНАЛІЗУ МАТЕМАТИЧНИХ ВЗАЄМОЗВ'ЯЗКІВ МІЖ КРИТЕРІЯМИ IRR, MIRR, PI

У статті здійснено аналіз переваг та недоліків показників внутрішньої норми прибутку та модифікованої внутрішньої норми прибутку як критеріїв ефективності інвестиційного проєкту. У процесі аналізу найважливіших показників ефективності інвестиційного проєкту використано математичний та графічний апарат для дослідження функцій і залежності між економічними характеристиками майбутньої фінансово-виробничої та підприємницької діяльності. Характеристики критерію внутрішньої норми прибутку досліджено на основі властивостей функції, яка описує залежність чистої теперішньої вартості проєкту від величини ставки дисконту. Виявлено основні переваги модифікованої внутрішньої норми прибутку порівняно з її немодифікованим аналогом. Математично доведено нерівності між зазначеними показниками, проаналізовано їхній уплив на характеристики стійкості (безпеки) інвестиційного проєкту та показано взаємозв'язок критерію модифікованої внутрішньої норми прибутку зі значенням індексу прибутковості. Наукова новизна полягає в математичному доведенні взаємозв'язку між показниками внутрішньої норми прибутку та модифікованої внутрішньої норми прибутку, а також взаємозв'язку між критерієм модифікованої внутрішньої норми прибутку та індексом прибутковості інвестиційного проєкту. Теоретичні висновки та пропозиції можуть бути використані при інвестиційному аналізі майбутніх фінансово-виробничих проєктів в економіці, що відкриває можливості раціонального використання ресурсів у підприємницькій діяльності на всіх рівнях управління бізнес-процесами.

Ключові слова: інвестиційний проєкт, індекс рентабельності, чиста приведена вартість, нестандартний грошовий потік, інвестиція

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