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THE RESEARCH OF INDUSTRIAL PRODUCTION DYNAMICS BASED ON THE TOOLS OF CHAOS THEORY

Purpose. To prove the possibility of improving the procedure for analyzing and forecasting the dynamics of economic systems through the comprehensive use of scientific achievements of chaos theory, namely: checking the trend stability of time series, studying their phase space, attractors, Lyapunov's chaos indicators, the maximum length of a reliable forecast of the socio-economic system development, etc.

Methodology. The methodological basis of the study is the provisions of modern economic theory, in particular, statistics, economic and mathematical modeling and forecasting, economic cybernetics and systems theory, fundamental works of foreign and domestic scientists on the issues of fractal analysis and chaos theory.

Findings. The phase and fractal analysis of the dynamics series of chain and basic growth rates of industrial production in Ukraine was carried out, and their fractal dimension was determined. The correlation function was calculated and Lyapunov's indicators were found to assess the degree of chaotic system, Kolmogorov entropy, and the parameter of evolution in time. The maximum length of a reliable forecast and the future values of the time series were also determined.

Originality. The article substantiates the necessity and possibility of applying the methodological apparatus of chaos theory in the process of analyzing and forecasting economic dynamics, including the development of domestic industrial production.

Practical value. The value of the work is determined by the applied aspects of reliable forecasts of chain and basic growth rates of industrial production in Ukraine obtained on the basis of the chaos theory tools, the possibility of comparative analysis of the domestic industry development in "potential peacetime" and actual wartime.

Keywords: *nonlinear dynamic systems, chaos theory, economic dynamics, persistence of time series*

Introduction. Despite the fact that the basic ideas of studying the nonlinear dynamics of objects of any nature within the framework of chaos theory appeared in the middle of the twentieth century (H. Hurst, B. Mandelbrot, E. Lorenz), their theoretical and methodological substantiation and practical use were mainly aimed at building and studying economic and mathematical models with continuous time, based on differential equations. Among them there are such well-known developments as macroeconomic models by Forrester, Solow, Walras, Keynes, and others. As for discrete-time models based on time series and difference equations, they have received much less attention from scientists of this direction.

At present, in the course of analysis and forecasting of economic dynamics discrete indicators at all levels of economic entities' management, the traditional paradigm prevails in the form of a normal distribution model of the studied attributes. Within the framework of this model, such indicators of economic dynamics as absolute growth, growth and increment rates, average development characteristics are calculated. Furthermore, current averages, correlation-regression and autocorrelation analyses, time series extrapolation, and statistical hypothesis testing procedures are applied. At the same time, the persistence of the time series is not checked, and the length of the anticipation (forecasting) period is determined solely on the basis of the normal distribution apparatus.

Literature review. The first works in the field of studying economic time series using the chaos theory apparatus date back to the second half of the twentieth century and were devoted mainly to the fractal analysis of financial markets. The most famous among them are the publications by B. Mandelbrot, E. Feder, E. Peters.

Their modern followers include the majority of domestic researchers, in particular: E. Naiman [1], K. Krytsun [2, 3], L. Nych, R. Kaminsky [4], S. Garder, T. Kornil [5], A. Kudzinowski, A. Morozyuk [6], and others. The mathematical and statistical apparatus of fractal analysis is described in detail by L. Kirichenko and T. Radivilova [7].

As for the works in the field of using the methodological apparatus of chaos theory in the process of analyzing and forecasting the dynamics of industrial production time series, it is necessary to note the works by O. Yankovyi, O. Goncharenko [8], I. Tchaikovska [9], O. Yastremska, G. Demchenko [10], O. Demydenko [11], O. Kryvda, Y. Sydorenko, D. Romanova

[12], O. Brovarets, Y. Chovniuk [13], V. Skalozub, V. Goryachkin, I. Klymenko, D. Shapoval [14], and others.

Tan H. [15] proposed to use the methodology of chaotic genetics in combination with a fuzzy decision algorithm as a result of the predictive analysis of time series in the economy. He notes that a chaotic series of dynamics has a certain internal regularity, which is manifested in the ratio of development cycles depending on the time delay. In his opinion, the results of the experiments show that the chaotic genetics algorithm combined with fuzzy decision-making can dynamically respond to chaotic mutations and generalize the experts' previous experience. The phase space of the reconstructed chaotic attractor has high-precision predictability and can determine the ordered processes of change in research results, which, in turn, are suitable for use in the analysis and forecasting of complex economic chaotic time series.

According to O. Bilska [16], the construction of pseudo-phase portraits in the process of analyzing and forecasting nonlinear economic systems allows the researcher to determine the presence of cyclical development, to establish the nature of its changes, as well as the prevailing trends in the studied indicators dynamics.

The article by O. Sergienko, M. Mashchenko, V. Baranova [17] proposes a comprehensive toolkit of models for studying the instability of development trajectories of complex hierarchical systems, which makes it possible to draw conclusions about the causes and factors of endogenous (self-generated) fluctuations and bifurcations, about the possibility of disasters and crises in complex hierarchical economic systems. Solving the problems caused by the instability of the complex hierarchical systems development based on the integrated application of phase, cointegration and bifurcation analysis allows one to predict crisis situations in advance and to apply methods of their prevention, to determine complex ways out of crisis situations.

The work by V. Solovyov, O. Serdyuk, S. Semerikov, and A. Kiva [18] is devoted to the possibility to analyze the dynamics of changes in the time series characteristics obtained on the basis of information about critical phenomena in economic systems. The characteristic of identifying critical phenomena was evaluated based on the study of a number of time series indicators, including the self-similarity coefficient, predictability coefficient, entropy, and laminarity. It is shown that the entropy analysis of a dynamics series in phase space reveals the characteristic properties of an economic system with recurrent crises. It is proved that the methodology of using the recurrence entropy in the study of critical phenomena has an advantage over the traditional entropy indicator use. It has been found that the recurrence entropy, unlike other indicators of complexity entropy, is an indicator and an early precursor of crisis phenomena in the economy.

Unsolved aspects of the problem. Modern authors of economic research practically do not discuss such applied tools for analyzing and forecasting time series dynamics at the micro, meso and macro levels as a phase portrait, attractor, maximum value of a reliable forecast, Lyapunov's chaos indicators in a socio-economic system, etc. Without consideration of these important concepts and indicators of chaos theory, studies of the socio-economic systems dynamics remain significantly limited and imperfect.

Purpose. The purpose of this paper is to prove that it is possible to improve the procedure for analyzing and forecasting the economic systems dynamics by making full use of the scientific achievements of chaos theory, in particular, checking the trend stability of time series, studying phase space, attractors, Lyapunov's chaos indicators, the maximum length of a reliable forecast, Kolmogorov entropy, and the parameter of evolution in time of an economic system. These issues are important for the formation of an adequate methodological toolkit of mathematical and statistical research that determines the effective management apparatus of the national economy.

Research methodology. The methodological basis of the article is the provisions of modern economic theory – statistics, economic and mathematical modeling and forecasting, economic cybernetics and systems theory, fundamental works of foreign and domestic scholars on fractal analysis and chaos theory.

Presentation of the main research material. Classification of dynamic systems in the economy. Currently, there is no generally accepted grouping of economic systems from the standpoint of the chaotic dynamics theory. In our opinion, to solve this problem, it is advisable to use a cybernetic approach to the dynamic systems classification of any nature – social, economic, information, physical, chemical, biophysical, environmental, etc. (Table 1).

Thus, the possibility to self-organize a dynamic system in the economy characterizes its ability to change both the development program (plan) and the internal structure (organizational, production, managerial). In the self-organization process the behavior of the economic system can change quite dramatically under the influence of endogenous and exogenous factors and conditions, and it itself belongs to the group of systems with catastrophic transformations. For example, a complete re-profiling of an industrial enterprise due to a sharp change in demand for certain types of products (works, services). Economic systems that do not have such catastrophic zones of behavior are called laminar (flexible).

All economic systems are open as they actively interact with the external environment by "consuming" production factors-resources (labor, capital, land, information, energy, achievements of science and technology, etc.) and satisfying the market demand for the corresponding products (works, services). There are practically no closed systems in the modern economy – they are actually open systems in which external flows of inputs and outputs are insignificant and can be neglected.

Open economic systems that are far from equilibrium and where there is an increase in entropy are called dissipative. The energy of ordered motion is dissipated in dissipative systems, i.e., it turns into the energy of disordered (chaotic) motion. This phenomenon is called "dissipation". It is often accompanied by coordination effects when the elements of the system correlate and coordinate their behavior. Such cooperative, coordinated behavior is characteristic to systems of various types, including living beings, economic systems, and social groups. In conservative (Hamiltonian) systems, it is usually assumed that there is no dissipation of energy, so from this point of view, almost all economic systems are dissipative.

According to the nature of the variables dynamics all systems are divided into groups with continuous and discrete variation of features over time. In principle, all real dynamic systems observed in nature and society varies continuously over time. However, when measuring and modeling them, a researcher often applies a discrete approach to the study of certain systems, including economic ones. For example, reporting on technical and economic indicators of an industrial enterprise for a year, quarter, month, etc. Therefore, in our opinion, it is appropriate to speak of classifying not the systems themselves but their models.

Table 1

Dynamical systems classification of any nature from the standpoint of the chaotic dynamics theory

Characteristic	Groups of systems
1. Ability to self-organize	Catastrophic; laminar
2. Interaction with the external environment	Open; closed
3. Energy dissipation in the system	Dissipative; conservative
4. The nature of the variables dynamics	Continuous; discrete
5. Structural design	Simple; hierarchical
6. The nature of the system dynamics	Determined; stochastic
7. Type of mathematical process of evolution	Linear; nonlinear

In addition, it is believed that the vast majority of economic systems are characterized by the stochastic influence of factors on the outcome indicators and this influence itself is described by nonlinear equations.

The procedure for applying the chaos theory tools in the study of economic dynamics series. The peculiarity of modern economic science is the comprehensive search for interpenetration and application of the achievements of various technical, mathematical and natural disciplines. A striking example of such interpenetration is the application of the chaos theory tools, which are an integral part of economic cybernetics, to the analysis and forecasting of economic indicators time series, related to the theory of statistics. In this article, the study of the chaotic properties of the time series was carried out on the example of the industrial production dynamics of Ukraine for 2015–2021, presented in the form of chain (*Gr ch*) and basic (*Gr bas*) growth rates [19]. It should be noted that there is a relationship between these indicators

$$Gr\ bas_n = Gr\ ch_1 \times Gr\ ch_2 \times \ldots \times Gr\ ch_n, \qquad (1)$$

that is, the product of successive chain growth rates is equal to the corresponding basic growth rate. Fig. 1 shows the chain and basic growth rates dynamics of industrial production in Ukraine in 2015–2021.

Both time series contain 84 observations each ($n = 12 \times 7$), so the sample is quite representative. We deliberately did not consider the data for 2022–2023, as they were obvious anomalous observations that reflected the force majeure conditions of the Ukrainian industry in the context of full-scale aggression by the RF. A visual analysis of the graphs in Fig. 1 shows a rather volatile nature of the industrial production dynamics over the studied period.

It should be noted that the procedure of applying the chaos theory tools in the analysis and forecasting of economic dynamics series usually includes the following main stages:

1. Studying the dynamics series in phase space (construction of pseudo-phase and phase portraits).
2. Checking the persistence of the time series by calculating the *H*-index of Hurst and determining the fractal dimension and correlation function of the time series.
3. Calculating Lyapunov's indicators, which reflect the sensitivity degree of the process to changes in initial conditions.
4. Determining the maximum length of a reliable forecast.
5. Forecasting future values of the time series.
6. Applying Lyapunov's indicators to determine the Kolmogorov entropy and phase volume compression as a characteristic of the economic system stability.

Let us begin studying the chaotic properties of the time series of domestic industrial production by constructing its pseudo-phase and phase portraits.

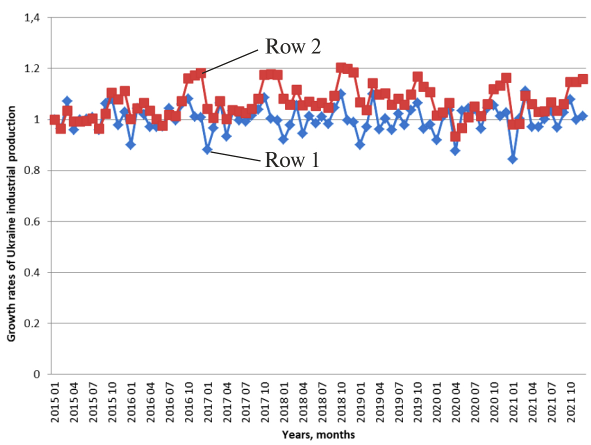

Fig. 1. Monthly chain (row 1) and basic (row 2) growth rates of Ukrainian industrial output for 2015–2021

A pseudo-phase portrait of the studied time series is an image in the Cartesian coordinate system of the dependence of its levels on the same series, which has a certain delay, i. e., with a time lag τ.

Based on formula (1) the time series of the basic growth rate can be viewed as the outcome variable *Y* compared to the time series of the chain growth rate *X*, which to some extent is a factor variable.

In Figs. 2, 3 for the variables *X* and *Y*, we show the autocorrelation fields and the corresponding autoregressive equations of the first order ($\tau = 1$), which were constructed using the STATISTICA program graphical tools, version 10.

For the variable *X* the first-order autocorrelation coefficient is -0.11 and for the variable *Y* it is 0.60. This means that the chain growth rates of industrial output are practically independent of previous values while for the basic growth rates there is a certain direct correlation with the past, whose density of which is 0.6. The fuzzy pseudo-phase portrait in Figs. 2, 3 is observed due to the fact that the studied economic system of industrial production quickly "forgets" its initial conditions of functioning. With a further increase in the time lag τ the appearance of the pseudo-phase portrait did not change – its blurring remained approximately the same.

A phase portrait is a representation of the trajectory of a dynamic economic system over time in phase space, i. e. in the coordinates of its variables (in this case in the *X, Y* coordinates on the plane). The state of the system in each period or mo-

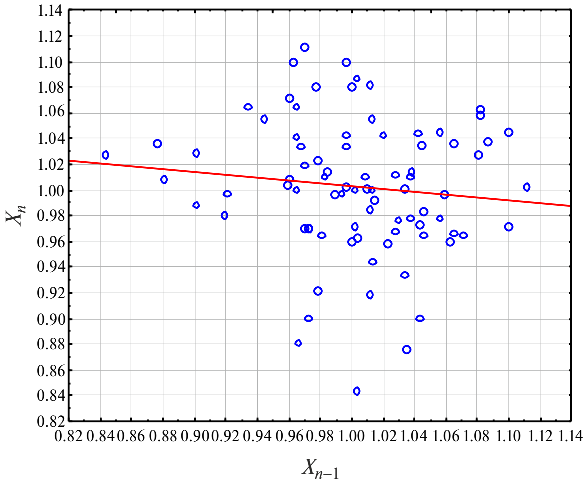

Fig. 2. Autocorrelation field and autoregression equation of the first order $X_n = 1.1136 - 0.1102X_{n-1}$

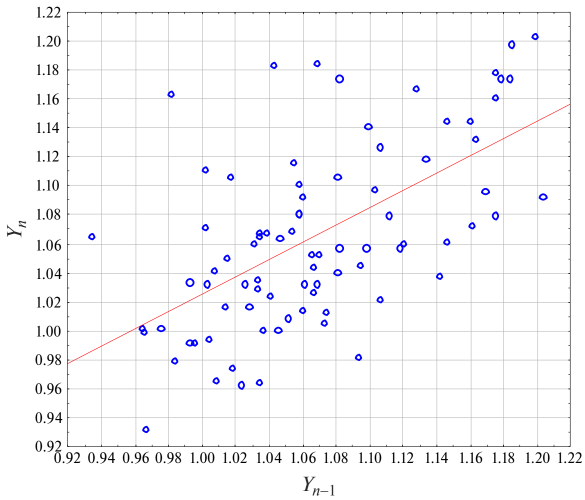

Fig. 3. Autocorrelation field and autoregression equation of the first order $Y_n = 0.4259 + 0.6021\,Y_{n-1}$

ment of time corresponds to a certain point on the phase portrait, which serves to visually display the evolution of a dynamic economic system: stationary points, cycles, areas of attraction (attractors). For a two-dimensional coordinate system X, Y, the phase portrait fully reflects all types of trajectories that have been realized during the time under study.

Fig. 4 presents a phase portrait of the first 14 points of industrial production dynamics in the X, Y coordinate system for simplicity and clarity. The initial point was the one with unit coordinates (marked 1), to which there is a certain attraction of other points – the industrial production growth rates. This means that the development process is characterized by cyclic behavior and indicates the presence of an attractive set – a point attractor, whose structure reflects the fractal nature of the economic system under study.

Indeed the cyclical behavior of the industrial production process is quite clearly manifested with a periodicity of 12 months when a more detailed consideration of the series of the chain growth rates dynamics X is taken (Fig. 1).

The persistence (trend stability) of the time series is checked by calculating the H-index of Hurst, which is carried out by two main methods: 1) on the basis of the accumulated normalized range of observations R/S; 2) through the value D of the fractal dimension of the dynamics series space.

The first method was proposed by H. Hurst himself, who, as a result of studying many natural processes, established an empirical relationship between the accumulated normalized range of the time series R/S and the length of the entire time interval through the index H

$$R/S=(aN)^H, \tag{2}$$

where S is the standard deviation of the observations accumulated series; N is the number of observation periods ($N>1{,}000$); a is a constant (for short time series, $a=0.5$).

Logarithmizing the left and right sides of (2), we obtain

$$H=\frac{\ln(R/S)}{\ln(aN)}. \tag{3}$$

Naiman E. [1], who studied the persistence of financial markets, proposed the following formula to calculate the H index for small samples

$$H=\frac{\log(R/S)}{\log(0.5\pi N)}(1.0136-0.0011\ln N). \tag{4}$$

Here are the main properties of the Hurst index:

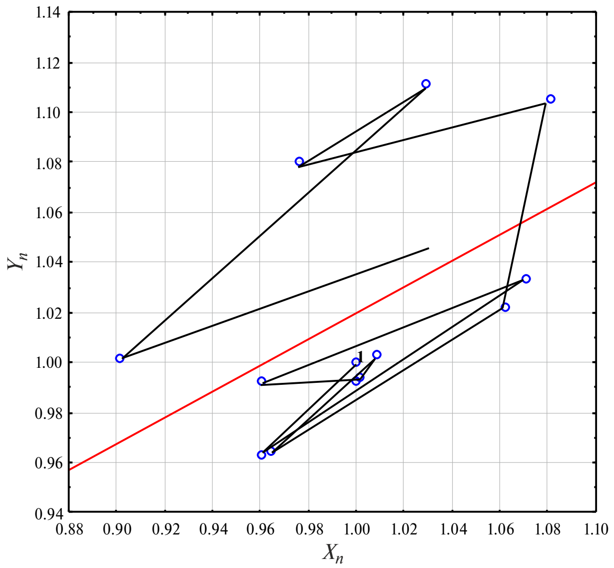

Fig. 4. The first 14 points of the phase portrait of the industrial products dynamics in the coordinate system X, Y

1) the value of H takes on values from 0 to 1 and depends on the dynamics series length: the longer it is, the more accurate the value of this indicator is;

2) for natural objects H values are grouped around 0.72–0.73;

3) if $H>0.5$ for time series, they are considered stable, trend-stable, i. e., those that maintain the existing trend (if the time series has grown in the past, it is likely to grow in the future and vice versa);

4) when $H=0.5$, the time series realizes a random process;

5) with $H<0.5$, time series are considered anti-permanent, unstable, i. e., the current trend tends to reverse;

6) the value of $H>1$ is very rare and occurs in the presence of independent abnormal jumps in the time series values;

7) Hurst index is related to the fractal dimension of the space D as follows

$$D=2-H. \tag{5}$$

From the relation (5), it follows that $H=2-D$, which makes it possible to implement the second method of estimating the Hurst index [4].

Formula (4) was used in this article to calculate the value of H, which characterizes the persistence degree of the studied dynamics series of Ukraine's industrial production, presented in the form of chain and basic growth rates, since the series length was only 84 observations. The calculations resulted in a value of $H=0.6796>0.5$. Its value indicates that the dynamics of Ukraine's industrial output, which is described by the time series of chained growth rates, is characterized by long-term memory and is trend-stable.

One of the important criteria for the system chaotic behavior is the fractality of the pseudo-phase portrait of the studied dynamics series, which is manifested in the value of its fractal dimension of space D. For self-similar processes there is a connection between the H-index and the fractal dimension of space D in the form of expression (5). Its value reflects the complexity of the analyzed time series structure and is in the range from 1 to 2, which directly follows from the limitations of the change in the Hurst index. In the task under discussion $D=2-0.6796=1.3204$. Thus, the application of fractal analysis to phase transitions allows us to quantify such interrelated properties of economic systems as the development trend and self-similarity of the structure of the process under study.

If a time series has the property of fractality (as in this case) its correlation function C is calculated based on the following formula

$$C=2^{2H-1}-1. \tag{6}$$

At the same time, it is time-independent and allows forecasting a series of dynamics for a sufficiently large number of steps forward. However, in practice, time series are only approximately fractal, so the lead time is always bounded from above. If the correlation function $C>0$ as in the task under discussion ($C=0.2827$), this indicates the persistence of the economic system dynamics since the index $H>0.5$. In the case of $C<0$ there is an anti-persistent development behavior ($H<0.5$) and when $C\approx 0$ the dynamics series implements a random process ($H\approx 0.5$).

Calculation of Lyapunov's indicators. To assess the degree of chaos in a system represented by time series it is necessary to determine its sensitivity to changes in initial conditions. As is known, this sensitivity is an attribute of the chaotic nature of any nonlinear dynamic systems, including economic ones. This property is manifested in the fact that the trajectories of two independent processes of a chaotic system in phase space are initially close to each other, but subsequently diverge exponentially. During the time t_r, which is called the time of forgetting the initial conditions, the information about the initial conditions in the system is completely lost. One of the main characteristics of such "memory loss" degree of the system is the Lyapunov's indicator.

If we consider the trajectories of two independent processes $x_1(t)$, $x_2(t)$ of a nonlinear dynamic system, the distance $d(t)$

between initially close trajectories increases over time according to the following formula

$$d(t) = |x_1(t) - x_2(t)| \approx \exp(\lambda t). \quad (7)$$

The value of λ is called Lyapunov's indicator and characterizes the change rate of the initial conditions of the process under study. For time series there is one Lyapunov's indicator for each of the phase space dimensions, for example, for the task under discussion these are *X, Y, t*. Therefore, we can assume that there are three Lyapunov's indicators for them: positive, zero, and negative. The positive indicator characterizes the stretching of the phase space, i. e. the speed of close points to move away. The negative indicator reflects the compression, i. e., the rate at which the system recovers from a perturbation. The zero value of λ indicates the temporary absence of chaotic properties of the system under study.

From formula (7) it follows that

$$\lambda t \approx \ln|x_1(t) - x_2(t)|. \quad (8)$$

To find one of Lyapunov's indicators λ, it is enough to construct a least-squares regression equation

$$\widehat{Y} = \lambda t + b, \quad (9)$$

where Y is determined by the right-hand side of expression (8). That is, λ is essentially the tangent of the angle φ of the regression line slope (9) to the t-axis. For a sharp angle of inclination φ ($0 \le \varphi < 90°$) the positive Lyapunov's indicator increases from 0 to $+\infty$ (at 90° it is undefined), i. e., a steep difference in the trajectories of the processes $x_1(t)$, $x_2(t)$ is manifested when $\lambda \to +\infty$. For an obtuse angle of inclination $\varphi(90° < \varphi \le 180°)$ the negative Lyapunov's indicator decreases from $-\infty$ to 0, i. e., a sharp convergence of the processes trajectories $x_1(t)$, $x_2(t)$ appears at $-\lambda \to 0$.

Thus, a positive slope of the regression line (9) to the time axis t indicates the presence of chaotic fluctuations in the behavior of the economic system, and a negative slope indicates regular processes in it (seasonal, cyclical, etc.).

For a pair of trajectories, for example, for the system of industrial production – agricultural production in Ukraine, it is necessary to choose a time interval that allows for separate calculation of both positive, zero, and negative Lyapunov's indicator (Fig. 5).

The system of straight lines shown in Fig. 5 has three Lyapunov's indicators. The straight dashed lines corresponding to the regression equations (9) were found by the least squares method. On a logarithmic scale, their positive slope (-----) gives $\lambda_1 = 0.227 > 0$ and their negative slope (— —) gives $\lambda_2 = -0.764 < 0$. There is another Lyapunov's indicator corresponding to an almost horizontal line – $\lambda_0 \approx 0$.

One of the most important results of chaos theory is the conclusion that the length of the forecast time horizon is fundamentally limited even for relatively simple systems. This applies

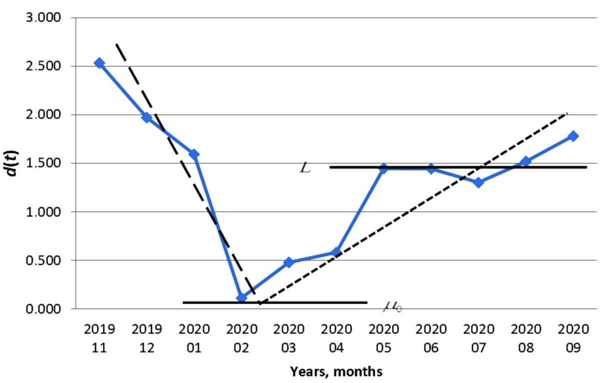

Fig. 5. To the calculation of Lyapunov's indicators and the maximum length of a reliable forecast for the basic growth rate of the industrial production system – agricultural production of Ukraine

to systems that are sensitive to initial conditions. The value of the positive Lyapunov's indicator characterizes the predictability horizon – the time period for which a reliable forecast of the behavior of the economic system under study can be provided.

The maximum length of a reliable forecast is determined by the forgetting time t_r of the dynamic economic system initial conditions, which is calculated using the following formula

$$t_r = \frac{\ln\left(\frac{L}{\mu_0}\right)}{\lambda_1}, \quad (10)$$

where μ_0 is the initial phase volume; L is the boundary value of the phase volume; λ_1 is the positive Lyapunov's indicator.

The initial and boundary values of the phase volume are determined based on the following considerations. The smallest value of $d(t)$ preceding the process of stretching two initially close phase trajectories is chosen as μ_0. To calculate the boundary value of the phase volume L, the interval of the series is determined where the value of $d(t)$ reaches a plateau corresponding to $\max d(t)$ (Fig. 5 for the horizontal lines μ_0, L).

In this task $\mu_0 \approx 0.081$; $L \approx 1.493$; $\lambda_1 = 0.227$ (Fig. 5) whence according to formula (10) we find $t_r = 12.837 \approx 13$. Thus, the calculated maximum time for a reliable forecast for the industrial production system – agricultural production of Ukraine for a series of dynamics of basic growth rates is 13 months. Similar calculations for the chain growth rates allowed us to determine $t_r = 9.274 \approx 9$ months.

The forecasting of future values of the time series was carried out on the basis of the stepwise Hurst method. The algorithm for determining the next predicted point of the time series consists of searching through all possible variants of future values of X_{n+1} (Y_{n+1}), constructing new time series containing $n + 1$ observations and calculating Hurst H-index for them. The point from the set of possible values that ensures the fulfillment of the condition $\min| H_n - H_{n+1}|$, where $H_n = 0.6796$ is the Hurst index, which was determined for the original time series of length $n = 84$, is chosen as the forecast value.

This procedure is cyclically repeated t_r times: 9 times for the time series of chain growth rates of industrial production, and 13 times for the basic growth rates. In the process of step-by-step calculations of the forecast values of the chain and basic growth rates of Ukraine`s industrial production for 9 and 13 months, respectively, we used the formula (4), which was used to determine the Hurst H-index for the original time series of $n = 84$.

Table 2 shows the projected values of the chain and basic growth rates of Ukraine's industrial production.

In our opinion, it is of some practical and political interest to compare the forecast values of the chain and basic growth rates of domestic industrial production based on trend-stable estimates of the development of one of the most important sectors of the Ukrainian economy in relatively peacetime with the actual values of the studied indicators for the same period the decline of which was caused by the full-scale aggression of the RF (Table 3).

A comparison of Tables 2 and 3 shows that starting in March 2022 there has been a significant decline in the basic (January 2015 = 100 %) growth rate of industrial production in Ukraine. Thus, the ratio of basic industrial production growth rates for a potentially "peaceful" and actual "military" January 2023 is 105.3 to 59.9 %. This means that, as a result of the war imposed on our country, industrial production at the beginning of 2023 was less than 59 percent of the level of early 2015.

The use of Lyapunov's indicators to determine the Kolmogorov entropy is another important direction in the dynamics series study of a dissipative economic system, characterizing the chaotic motion in the phase space of any dimension.

Kolmogorov entropy K_0 is calculated as the sum of positive Lyapunov's indicators. For dynamics series with only one posi-

Table 2

Forecast values of chain and basic growth rates of industrial production in Ukraine for January 2022 – January 2023, %

Forecast growth rates	2022 01	2022 02	2022 03	2022 04	2022 05	2022 06	2022 07	2022 08	2022 09	2022 10	2022 11	2022 12	2023 01
Chain	89.43	99.11	106.57	94.51	100.41	99.46	102.21	98.07	104.47	–	–	–	–
Basic	103.16	101.75	108.40	102.47	102.83	102.19	104.43	103.43	106.96	115.39	114.80	115.47	105.31

Table 3

Actual values of chain and basic growth rates of industrial production in Ukraine for January 2022 – January 2023, % [19]

Actual growth rates	2022 01	2022 02	2022 03	2022 04	2022 05	2022 06	2022 07	2022 08	2022 09	2022 10	2022 11	2022 12	2023 01
Chain	87	87.4	57.2	111.9	107.5	100.3	100.6	101.9	102.3	105	99.3	96.3	93.5
Basic	100.90	88.19	50.44	56.45	60.68	60.86	61.23	62.39	63.83	67.02	66.55	64.09	59.92

tive Lyapunov's indicator, $K_0 = \lambda_1$. The value of K_0 indicates the degree of chaotic movement of the economic system: chaotic movement when $K_0 > 0$, random movement when $K_0 \to +\infty$.

In this task $K_0 = \lambda_1 = 0.227$, i.e., there was a certain chaotic movement of the industrial production system – agricultural production of Ukraine in 2015–2021.

Based on the Kolmogorov entropy K_0 we can determine the metric entropy of the system S

$$S = K_0 t, \tag{11}$$

which value is proportional to the rate of information loss about the state of a dynamic economic system over time. In the example under discussion for the studied system $S = 0.227t$.

Compression of the phase volume as a characteristic of the state of the economic system stability. In chaotic systems in contrast to conservative ones the phase volume changes over time, i.e., the phase volume shrinks to a certain minimum value. For such systems, there is a relationship between the Lyapunov's indicators and the phase volume evolution parameter $V(t)$ through the characteristic time of its change process v_0, which is equal to the inverse of the sum of all λ_i. Since there are three Lyapunov's indicators for the dynamics series, we can write

$$\nu_0 = \frac{1}{\lambda_1 + \lambda_0 + \lambda_2}. \tag{12}$$

Taking into account (12), the expression for the phase volume evolution parameter $V(t)$ takes the form

$$V(t) = \mu_0 \exp\left[(\lambda_1 + \lambda_0 + \lambda_2)t\right]. \tag{13}$$

For economic chaotic systems represented by dynamics series, at $\sum \lambda < \quad v_0 < 0$ and the time parameter $t < 0$.

It follows from formula (13) that in this case the phase volume is compressed. If the value of the parameter $V(t)$ tends to zero over time then the economic system under study is striving to reach a steady state.

With an increase in the parameter $V(t)$, when the phase volume tends to approach a certain constant, the system is chaotic and has several equilibrium points. If, on the other hand, the phase volume increases continuously over time, the process becomes more and more random.

In the task under discussion

$$\sum_{i=} \lambda_i = 0.227 + 0 - 0.764 = -0.537 < 0.$$

From formula (12) follows that $v_0 = -1.862 < 0$ and the phase volume of the industrial production system – agricultural production of Ukraine is compressed. At the same time, according to (13) the parameter of evolution of the system phase volume is described by the following expression

$$V(t) = 0.081 \exp(-1.862t),$$

and decreases exponentially. This makes it possible to determine the value of the phase volume parameter in each specific month of the study period.

Thus, in January 2015 ($t = 1$) the phase volume parameter of the system under study was approximately 0.01258, in January 2022 $V(85) = 1.48832\text{E}{-70} \approx 0$, and in January 2023 it was even smaller: $V(97) = 2.94321\text{E}{-80}$, i.e., it tended to zero.

This means that the studied system of industrial products – agricultural products of Ukraine is sustainable in the future.

Conclusions and prospects for further development in this direction. The analysis and forecasting of the industrial production dynamics in Ukraine carried out using the tools of chaos theory opened up new opportunities in the study of time series of industrial production. In particular, a classification of dynamic systems in the economy was carried out, which allowed us to position them as capable of self-adjustment and self-organization, open, dissipative with discrete variation of features over time and a hierarchical structure. Dynamic economic systems are characterized by a stochastic nonlinear influence of factors on the performance indicators.

The construction of pseudo-phase portraits of the industrial production dynamic system in Ukraine represented by the series of dynamics of chain and basic growth rates, allowed us to state their fuzziness due to the fact that the studied economic system quickly "forgets" its initial conditions of functioning. With a further increase in the time lag the appearance of the pseudo-phase portraits did not change. The visual analysis of the phase portrait shows that the development process is characterized by a certain cyclical behavior and indicates the presence of an attractive set – a point attractor that reflects the fractal nature of the studied economic system.

Checking the self-similarity of the industrial production system in Ukraine on the basis of fractal analysis showed that it is persistent (Hurst H value is 0.6796, the dimension of the phase space $D = 1.3204$, the correlation function $C = 0.2827$). This opened up the possibility of using the existing trend to forecast future values of chain and basic growth rates of industrial production.

The found Lyapunov's indicators were used to estimate the maximum length of a reliable forecast, which was 13 months for the base and 9 months for the chain growth rates. The forecasting of future values of time series using the step-by-step Hurst method allowed us to compare a potentially possible peaceful scenario for the development of one of the most important sectors of Ukraine's economy with the actual values of the studied indicators, the decline of which was caused by the aggressive military actions of the RF. The calculations showed that as a result of the war imposed on Ukraine, its industrial production at the beginning of 2023 was less than 59 percent of the level of early 2015.

In addition, Lyapunov's indicators were also used to determine the Kolmogorov entropy and characterize the sustainability of the industrial production system – agricultural products of

Ukraine. According to our calculations, the Kolmogorov entropy is equal to 0.227, i.e., there is a certain chaotic movement of the studied economic system and it tends to a steady state.

As a prospect for further development in this scientific direction, we see the further mastery and widespread use of the entire effective mathematical and statistical apparatus of chaos theory, in particular, attractors and bifurcation points. The use of these important concepts and indicators of the chaotic development theory of systems of any nature can significantly enrich the study of the economic objects dynamics at all levels of economic management.

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Дослідження динаміки промислового виробництва на основі інструментарію теорії хаосу

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Мета. Доведення можливості удосконалення процедури аналізу та прогнозування динаміки економічних систем за допомогою всебічного використання наукових здобутків теорії хаосу, а саме: перевірки трендостійкості часових рядів, дослідження їх фазового простору, атракторів, показників хаотичності Ляпунова, максимальної довжини достовірного прогнозу розвитку соціально-економічної системи та ін.

Методика. Методологічну основу дослідження становлять положення сучасної економічної теорії, зокрема, статистики, економіко-математичного моделювання та прогнозування, економічної кібернетики й теорії систем, фундаментальні праці зарубіжних і вітчизняних учених з питань фрактального аналізу й теорії хаосу.

Результати. Проведено фазовий і фрактальний аналіз рядів динаміки ланцюгових і базисних темпів росту промислового виробництва в Україні, визначена їх фрактальна розмірність, здійснено розрахунок кореляційної функції й знаходження показників Ляпунова задля оцінки ступеню хаотичності системи, ентропії Колмогорова, параметра еволюції в часі, а також визначена максимальна довжина достовірного прогнозу й самі майбутні значення часового ряду.

Наукова новизна. Полягає в обґрунтуванні необхідності й можливості застосування методологічного апарату теорії хаосу у процесі аналізу та прогнозування економічної динаміки, у тому числі, розвитку вітчизняного промислового виробництва.

Практична значимість. Визначається прикладними аспектами отриманих на базі інструментарію теорії хаосу достовірних прогнозів ланцюгових і базисних темпів росту промислового виробництва в Україні, можливістю порівняльного аналізу розвитку вітчизняної промисловості в «потенційно мирні» та фактично воєнні часи.

Ключові слова: *нелінійні динамічні системи, теорія хаосу, економічна динаміка, персистентність часового ряду*

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