ANALYSIS OF MATHEMATICAL MODELS DESCRIBING THE REQUIREMENTS FOR NETWORK DEVELOPMENT

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Abstract: *The analysis of mathematical models describing the requirements for network development is provided in this paper.*

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Introduction

Whatever purposes would be set when at the simulation of the process of network development, the requirements to the services provided by the network at every instant, and therefore requirements to change in network capacity shall be an obligatory element of the given data.

To determine the laws describing the processes of network development, depending on the specific conditions, various mathematical models are used.

Main part

Particularly, usage of the mathematical theory of struggle for existence is found as convenient for the description of variation the quantity of the users connected to the information network. Using the mathematical methods describing biological processes, offered in [Volterra, 1976] it is possible to consider stated task as follows.

For two mutually competing information networks providing delivery of the information accordingly to N_1 and N_2 users, we'll work out two differential equations, describing these networks' development.

$$\frac{dN_1}{dt} = \left[\varepsilon_1 - \gamma F(N_1, N_2)\right] N_1 \tag{1}$$

$$\frac{dN_2}{dt} = \left[\varepsilon_2 - \gamma F(N_1, N_2)\right] N_2 \tag{2}$$

Expressions in brackets represent coefficients of users' number's growth for each network, ε_1, ε_2 - intensity of development of considered networks, in relation to one user, γ_1, γ_2 - factors characterizing reduction of users' growth for each network, in relation to one user.

$F(N_1 N_2)$ - function showing requirement for services of considered networks, turning into zero simultaneously with sum $N_1 + N_2$ and monotonously aspiring to infinity together with each of these variables.

Receiving

$$F(N_1 N_2) = \lambda_1 N_1 + \lambda_2 N_2,$$

where λ_1, λ_2 - coefficients showing distribution of requirements in infocommunication services between two networks.

In the stationary case for users of one network we receive

$$N = \frac{\frac{\varepsilon_1}{\lambda_1\gamma_1}}{1+\frac{\lambda_1\gamma_1}{c}e^{\varepsilon_1(t-t_1)}} = \frac{a}{1+be^{-ct}}, \tag{3}$$

where

$$a = \frac{\varepsilon_1}{\lambda_1\gamma_1}; \quad b = \frac{\lambda_1\gamma_1}{c}; \quad c = \varepsilon_1.$$

The second network has zero limit of development and disappears at t →∞.

It is necessary to notice, that at the considered case the limit of development of information network represents logistical function for transitive period.

At simultaneous existence of two networks, it is possible to receive decisions not for common, but for special cases and these decisions differ from "pure" logistical function, (for example, at $\gamma_1 = \gamma_2$) the decision of a differential winding of Volterra equations gives

$$N_1 = \frac{\frac{N_1^0\varepsilon_1\varepsilon_2}{\gamma}e^{\varepsilon_1(t-t_0)}}{\varepsilon_2\lambda_1N_1^0\left[e^{\varepsilon_1(t-t_0)}-1\right]+\varepsilon_1\lambda_2N_2^0\left[e^{\varepsilon_2(t-t_0)}-1\right]+\frac{\varepsilon_1\varepsilon_2}{\gamma}}, \tag{3}$$

$$N_2 = \frac{\frac{N_2^0\varepsilon_1\varepsilon_2}{\gamma}e^{\varepsilon_2(t-t_0)}}{\varepsilon_2\lambda_1N_1^0\left[e^{\varepsilon_1(t-t_0)}-1\right]+\varepsilon_1\lambda_2N_2^0\left[e^{\varepsilon_2(t_2-t_0)}-1\right]+\frac{\varepsilon_1\varepsilon_2}{\gamma}}. \tag{4}$$

In general arrival of the requirements to increase the network capacity is a stochastic process. Nevertheless acceptable results can be obtained by creating a requirements model using the deterministic function of time. The investigation of the effect of stochastic requests on the problem of optimization of expansion of network capacity proved that the considered stochastic models of requests for connection to the network can be replaced by the corresponding deterministic model.

Such a model shall clearly define the requirements for network services at every instant.

In the vast majority of the works devoted to optimizing information network the model of the requirements to the network development is described by a linear function of time-type

$$D(t) = b + kt, k \neq 0$$

where b - shift parameter;

k - slope coefficient.

The shift parameter determines the value of the function in the point $t = 0$ and allows specifying a reference amount of the value to be determined. The slope coefficient, which is numerically equal to the tangent of the angle between the function graph and the x-axis, determines the rate of increase or decrease of the function. The functions of this type change strictly monotonously, with a constant speed, which directly follows from the constancy of the first derivative

$$\frac{dD}{dt} = k = \text{const}.$$

Constant and unlimited increase or decrease in function allows us to use this function to simulate the stable periods of network development, in particular, it may relate to the initial stage of development of the network when its capacity is increasing uniformly. Linear dependence can be used for a relatively short period of time.

Using a power function as follows

$$D(t) = t^{\alpha}, \quad \alpha \in \boldsymbol{R}, \alpha \neq 0, \alpha \neq 1$$

allows us to describe the process that is already changing unevenly and can be used to describe a wide range of processes, depending on the parameter α . In particular, if $\alpha > 1$ the function increases indefinitely with increasing speed and may have a number of points of extremum and inflection points. These functions are suitable to describe the process of rapid development, in which the number of users increases rapidly besides the further it goes the more intensive it becomes.

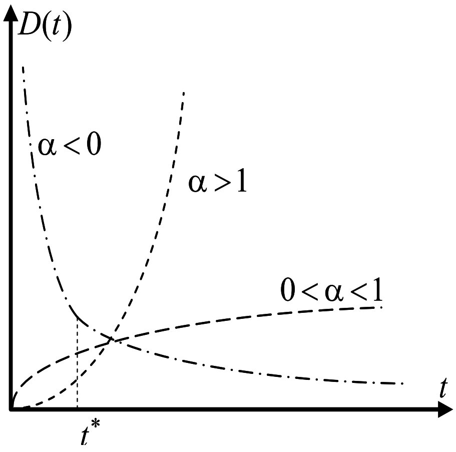

Fig. 1 Power function

When $0 \leq \alpha \leq 1$ a reverse process is observed. The function increases rapidly at the beginning, but with the increase of the argument value it slows down. However, one can not speak about the boundedness of the functions of this type. No matter how great value A is, the corresponding value of the argument will always be present t_A , $t_A = A^n : D(t_A) = t_A^{\frac{1}{n}} = \sqrt[n]{A^n} = A$. The functions of this type can be useful when simulating a long period of the network development, with a fairly rapid growth in the beginning and gradually slowing rate in the course of development.

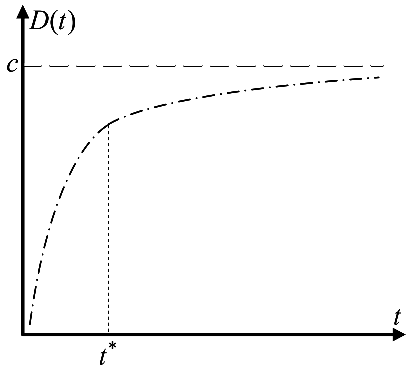

Fig. 2 Variation power function

When α values are negative, the power function will decrease. For sufficiently small t -values the values of the function can be arbitrarily large, the speed of decrease in such a section is also high. A similar situation is observed up to the "saddle point" t^* , approaching to which the magnitude and speed of decrease are gradually reducing and beyond which the function is decreasing rather slowly. As has already been noted, the function is unbounded in the sphere of its definition. One can also note its continuity and strict monotonousness. As it can be seen when $\alpha < 0$ the function behaves itself quite specifically and may be suitable only to describe the process of intense drop in demand for services. However, having provided the function with a negative coefficient, the opposite result can be achieved $\Theta(-t^{\alpha} + c)$, where c - is a positive number that determines the shift parameter. Thus it is convenient to set the initial capacity of the network before its modernization.

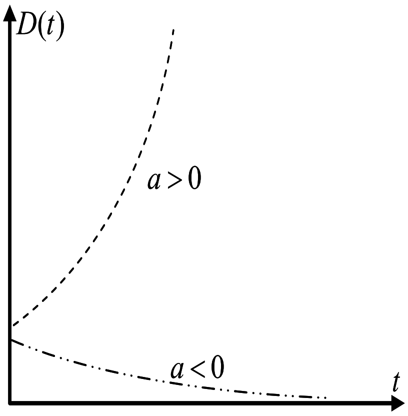

Fig. 3 Exponential function

A modified power function is already not decreasing but increasing, besides very rapidly at the beginning. After having passed the "saddle point" the growth becomes not so significant and with the increase of t it will be less and less noticeable. Despite some similarity in the behavior of the function under the question and t^{α} function upon $\alpha \in (0,1)$ their fundamental differences shall be noted.

When $t \to +\infty$ the function $D(t) = -t^{\alpha} + c$ has the following oblique asymptote $f(t) = at + b \equiv c$, with equality:

$$\lim_{t \to +\infty} \left[D(t) - (at + b) \right] = \lim_{t \to +\infty} \left[D(t) - c \right] = 0 .$$

Saturation of $D(t) = -t^{\alpha} + c$ function can be useful when simulating late or prolonged periods of the network development when the demands eventually come to a constant. Besides the intensity of function growth up to a saddle point emphasizes the acceptability of such functions to describe the prolonged development period, which is characterized by a rapid growth in the beginning and saturation at the end of the period studied.

Let`s analyze the possibility of using an exponential function as follows

$$D(t) = a^{t}, a > 0, a \neq 1 .$$

When $a > 1$ the function grows unboundedly monotonously and fast enough (faster than all hitherto examined functions) with an increasing rate of growth. The main differences from similar function $D(t) = t^{\alpha}$ when $\alpha > 1$ are higher speed of exponential function growth in comparison with the power function and the behavior at the beginning of a process that is when $t = 0$ the function a^{t} begins to grow with a significant speed, while the growth speed function t^{α} in the point $t = 0$ tends to zero

$$\frac{dD}{dt} = \alpha t^{\alpha - 1} \xrightarrow{t \to 0} 0 . \tag{5}$$

Except for the mentioned differences, the properties of the exponential function at $a > 1$ are asymptotically close to those of a power function, and the conclusions of the analysis of these functions are respectively similar.

The dependence (1) can not be used for unlimited period of time since D(t) tends to infinity. However, in many cases when the network capacity is small the exponential dependence of is a good model for forecasting purposes.

In the case when $0 < a < 1$ the function decreases continuously monotonically with decreasing speed of decrease.

We now turn to the logarithmic function

$$D(t) = \log_{a} t, a \in \boldsymbol{R}, a > 0, a \neq 1 .$$

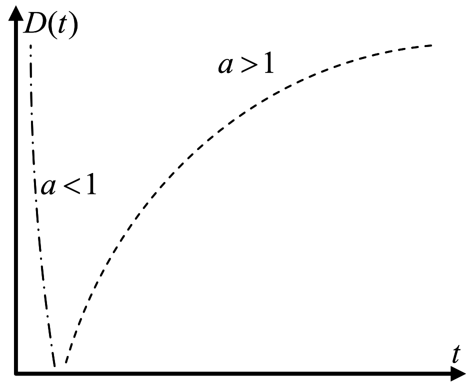

Fig. 4 Logarithmic function

When $a > 1$ the function $\log_{a} t$ is growing unboundedly monotonously and at $a < 1$ rapidly decreases. The growth speed in the first case is even lower than that of a linear function and the speed of decrease in the second one is extremely high.

Besides, $\log_{a} t$ with $a < 1$ is infinitely high at $t \to 0$.

Trigonometric functions occupy special place in this analysis. The main functions $\sin$, $\cos$, tg and ctg are periodic, which allows their use for simulating fluctuating processes of development. Both functions themselves and their composition with other functions can be used.

Thus, for example, the function of the type $\Theta(t + \sin t)$ can describe a process of continuous growth with periodically changing speed.

In telephony and similar spheres the assumption that requirements are a linear function of time is made, at that use of illustrative and logistics functions is possible.

The usage of the following logistic functions

$$Y = \frac{a}{1+be^{-ct}} ,$$

where Y - indicator of saturation;

t - current time coordinate;

a - limiting amount of saturation value;

b - coefficient characterizing the initial conditions of the process;

c - growth speed,

is convenient to describe the developing process, under constraints of growth.

If the absolute index of network growth is proportional to the achieved level of development and, at the same time, there are certain factors acting in the opposite direction, for example, wear and tear of existing hardware preventing its further use for the rendering of telecommunications services, this situation can be described by the expression

$$\frac{dY}{dt} = (\chi - \psi)Y , \tag{6}$$

where χ - coefficient determining the speed of growth of the network capacity;

ψ - coefficient including the speed of wear and tear of the existing hardware.

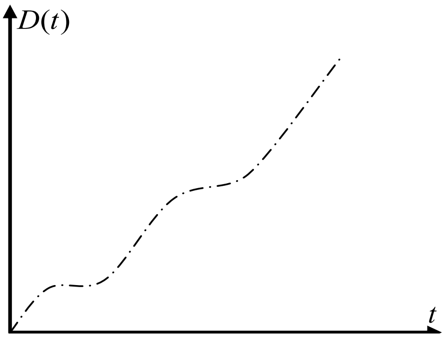

Fig. 5 Trigonometric functions

Assuming that χ = const, and $\psi = \psi_1 Y$, we will obtain

$$\frac{dY}{dt} = (\chi\chi\text{-}_1Y)Y = \chi\left[\frac{1\text{-}\frac{Y}{\chi}}{\psi_1}\right]Y = c\left(1-\frac{Y}{a}\right)Y , \tag{7}$$

where $\chi = c$

$a = \chi / \psi_1$.

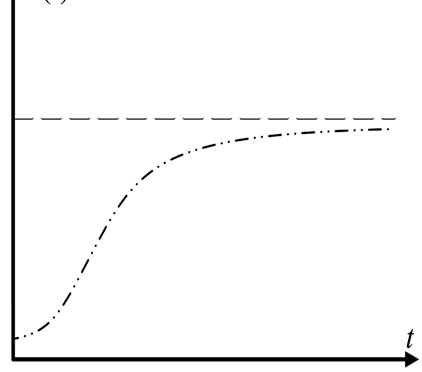

Fig. 6 Logistics function

In general, χ and ψ, can be both the functions of the achieved level of the network development and the functions of time. Upon the linear nature of these dependences, we have

$$\chi = \chi_0 + \chi_1 t + a_0 + a_1 Y , \tag{8}$$

$$\psi = \psi_0 + \psi_1 t + b_0 + b_1 Y , \tag{9}$$

where χ_0, χ_1, ψ_0, ψ_1 – coefficients characterizing the dependence of the functions χ and ψ on the time, and a_0, a_1, and b_0, b_1 - on the level of the network development.

Substituting the functions (8 and 9) in (6), we obtain

$$\frac{dY}{dt} = [(a_1\text{-}b_1)Y + (\chi_1\text{-}\psi_1)t + (a_0 + \chi_0) - (b_0 + \psi_0)]Y . \tag{10}$$

Introducing the denotations

$$\varepsilon = a_1 - b_1; \zeta = \chi_1 - \psi_1; \eta = (a_0 + \chi_0) - (b_0 + \psi_0) \quad (11)$$

expression (10) can be written as

$$\frac{dY}{dt} = \varepsilon Y^2 + \xi \, tY + \eta Y \quad (12)$$

as the result we obtain a well-known differential Riccardi equation of the type

$$\frac{dY}{dt} + f(t)Y + (t)Y^2 = 0, \quad (13)$$

where ε = 2,

$f(t)$ = - (ζ t + η);

$\varphi(t)$ = - ζ.

The solution of this differential equation gives

$$Y = \frac{e^{\frac{gt^2}{2}} + \eta t}{C - j\int e^{\frac{gt^2}{2}} + \eta t dt}, \quad (14)$$

where c - constant of integration.

The main features of the logistics process are:

- positive value of process characteristic at the initial moment of time;
- relatively rapid growth of curve in the initial stage of the process;
- presence of an inflection point;
- slow growth of the curve after the inflection point;
- asymptotic approximation process to the limit of saturation.

Conclusion

Logistical function is applied to the description of various processes, with accuracy, higher, than approximating functions that allows to apply the logistical model, offered Volterra to the mathematical description of struggle for existence of individuals in the nature, competing in struggle for natural resources, for forecasting of development of information networks.

However logistical function is not convenient for the decision of optimizing problems as does not reflect influence of external actions on a course of real processes.

Logistics development is possible only if the c and ψ-functions of the level of the network development, that is $c = f(Y)$; $\psi = f(Y)$. If at least one of these functions does not depend on Y, and is the function of time of the process development, the logistic process turns into ecological one, where withering is away takes place instead of saturation at the final stage of the network development.

The analysis performed allowed to use various functions describing the behavior of the requirements model for development of the networks for simulating various situations that occur on specific information network.

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