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ABSTRACTS

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Asymptotic Behavior of Solutions of Third Order Nonlinear Differential Equations Close to Linear Ones

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The differential equation

$$y''' = \alpha_0 p(t) y |\ln |y||^{\sigma} \tag{1}$$

is considered, where $\alpha_0 \in \{-1; 1\}$, $\sigma \in \mathbb{R}$, $p : [a, w) \to (0, +\infty)$ is a continuous function; $a < w \leq +\infty$.

Asymptotic properties of solutions of equation (1) when $\sigma = 0$ were investigated in detail in the work by I. T. Kiguradze [6, § 6]. For second order equations of the form (1) the asymptotic of solutions of this class was studied in the works by V. M. Evtukhov and Mousa Jaber Abu Elshour [1, 3].

In this work the equation of the third order equation (1) is investigated using the methodology proposed by V. M. Evtukhov for differential equations of n-th order in [2] and further developed in the works [4, 5, 9]. Some results for equation (1) we published in [7, 8].

The solution y of equation (1), defined on the interval $[t_y, w) \subset [a, w)$ is called $P_w(\lambda_0)$ solution if it satisfies the following conditions:

$$\lim_{t\to w} y^{(k)}(t) = \begin{cases} \text{either } \ 0, \\ \text{or } \ \pm\infty, \end{cases} \quad (k = 0, 1, 2), \qquad \lim_{t\to w} \frac{(y''(t))^2}{y'''(t) y'(t)} = \lambda_0.$$

Necessary and sufficient conditions for the existence of $P_w(\lambda_0)$ solutions of equation (1) are stated. The asymptotic representation of such solutions and their derivatives up to second order when $t \to w$ were received.

Let us introduce the necessary notation.

$$\pi_w(t) = \begin{cases} t & \text{if } w = +\infty, \\ t - w & \text{if } w < +\infty, \end{cases} \quad I_A(t) = \int_A^t \pi_w^2(\tau) p(\tau)\, d\tau, \quad I_B(t) = \int_B^t p^{\frac{1}{3}}(\tau)\, d\tau,$$

$$A = \begin{cases} a & \text{if } \displaystyle\int_a^w |\pi_w(\tau)|^2 p(\tau)\, d\tau = +\infty, \\ w & \text{if } \displaystyle\int_a^w |\pi_w(\tau)|^2 p(\tau)\, d\tau < +\infty, \end{cases} \qquad B = \begin{cases} a & \text{if } \displaystyle\int_a^w p^{\frac{1}{3}}(\tau)\, d\tau = +\infty, \\ w & \text{if } \displaystyle\int_a^w p^{\frac{1}{3}}(\tau) d\tau < +\infty, \end{cases}$$

$$q(t) = p(t)\pi_\omega^3(t) \big| \ln \pi_\omega^2(t) \big|^{\sigma}, \quad Q(t) = \int_a^t p(\tau)\pi_\omega^2(\tau) \big| \ln \pi_\omega^2(t) \big|^{\sigma}\, d\tau.$$

Let us formulate the main theorem on the existence of $P_w(\lambda_0)$ solutions of equation (1).

Theorem 1. *Let $\sigma \neq 1$. Then for the existence of $P_w(\lambda_0)$ solutions of equation* (1)*, where $\lambda_0 \in \mathbb{R}\setminus\{0; 1; \frac{1}{2}\}$, it is necessary, and if the function $p : [a, w) \to (0, +\infty)$ is continuous and differentiable and*

$$\lambda_0 \neq \frac{-(2+\sigma) \pm \sqrt{(2+\sigma)^2+8}}{4}, \quad \lambda_0 \neq \frac{-1 \pm \sqrt{3}}{2}, \quad \lambda_0 \neq \frac{-(2-\sigma) \pm \sqrt{(2+\sigma)^2+8}}{4},$$

then it is also sufficient that

$$\alpha_0\lambda_0(2\lambda_0-1)(\lambda_0-1)\pi_w(t) > 0, \quad \lim_{t\to w} \frac{p(t)\pi_w^3(t)}{\left|\frac{(1-\sigma)(1-\lambda_0)^2}{\lambda_0}\, I_A(t)\right|^{\frac{\sigma}{\sigma-1}}} = \alpha_0 \frac{|\lambda_0|\,|2\lambda_0-1|}{|\lambda_0-1|^3}\,. \tag{2}$$

Moreover, for each of such solutions there are asymptotic representation as $t \to w$

$$\ln|y(t)| = \nu\Big((1-\sigma)\frac{(\lambda_0-1)^2}{\lambda_0} I_A(t)\Big)^{\frac{1}{1-\sigma}}(1+O(1)),$$

$$\frac{y'(t)}{y(t)} = \frac{(2\lambda_0-1)}{(\lambda_0-1)\pi_w(t)}\,(1+O(1)), \quad \frac{y''(t)}{y'(t)} = \frac{\lambda_0}{(\lambda_0-1)\pi_w(t)}\,(1+O(1)),$$

where $\nu = \operatorname{sign}(\alpha_0(\lambda_0-1)(1-\sigma)I_A(t))$.

Theorem 2. *Let $\sigma \neq 3$. Then for the existence of $P_w(1)$ solutions of equation* (1) *it is necessary, and if $p : [a, w) \to (0, +\infty)$ is continuous and differentiable and such that there is a finite or equal $\pm\infty$*

$$\lim_{t\to w} \frac{(p^{\frac{1}{3}}(t)|I_B(t)|^{\frac{\sigma}{3-\sigma}})'}{p^{\frac{1}{3}}(t)|I_B(t)|^{\frac{3\sigma}{3-\sigma}}},$$

then it is also sufficient that

$$\lim_{t\to w} \pi_w(t)p^{\frac{1}{3}}(t)|I_B(t)|^{\frac{\sigma}{3-\sigma}} = \infty.$$

Moreover, for each of such solutions the are asymptotic representation as $t \to w$

$$\ln|y(t)| = \mu\Big|\frac{3-\sigma}{3}\, I_B(t)\Big|^{\frac{3-\sigma}{3}}(1+O(1)),$$

$$\frac{y'(t)}{y(t)} = p^{\frac{1}{3}}\Big|\frac{3-\sigma}{3}\, I_B(t)\Big|^{\frac{\sigma}{3-\sigma}}(1+O(1)), \quad \frac{y''(t)}{y(t)} = p^{\frac{1}{3}}\Big|\frac{3-\sigma}{3}\, I_B(t)\Big|^{\frac{\sigma}{3-\sigma}}(1+O(1)),$$

where $\mu = \operatorname{sign}(\frac{3-\sigma}{3}\, I_B(t))$.

Theorem 3. *For the existence of $P_w(\pm\infty)$ solution of equation* (1)*, necessary and sufficient conditions are:*

$$\lim_{t\to w} q(t) = 0, \quad \lim_{t\to w} Q(t) = \infty.$$

Moreover, for each of such solutions there are asymptotic representation as $t \to w$

$$\ln|y(t)| = \ln \pi_w^2(t) + \frac{\alpha_0 Q(t)}{2}\,(1+O(1)),$$

$$\ln|y'(t)| = \ln|\pi_w(t)| + \frac{\alpha_0 Q(t)}{2}\,(1+O(1)), \quad \ln|y''(t)| = \frac{\alpha_0 Q(t)}{2}\,(1+O(1)).$$

The asymptotic of solutions in Theorems 1–4 is written in implicit form. The conditions for the existence of solutions of equation (1) of the specified type were obtained in which their asymptotic performance, as well as derivatives of first and second order can be written in explicit form.

Theorem 4. *Let $\sigma(1-\sigma)\neq 0$ and conditions (2) take place. Let, in addition $\lambda_0\in\mathbb{R}\setminus\{0;1;\frac{1}{2}\}$, $\lambda_0\neq -1\pm\sqrt{3}$ and the functions*

$$h_1(t)=\frac{p(t)\pi_\omega^3(t)}{\Big|\frac{(1-\sigma)(1-\lambda_0)^2}{\lambda_0}I_A(t)\Big|^{\frac{\sigma}{\sigma-1}}}-\frac{\alpha_0|\lambda_0|\,|2\lambda_0-1|}{|\lambda_0-1|^3}\,,\quad h_2(t)=\Big|(1-\sigma)\,\frac{(\lambda_0-1)^2}{\lambda_0}\,I_A(t)\Big|^{\frac{1}{\sigma-1}},$$

such that

$$\lim_{t\to\omega}\frac{h_1(t)}{h_2(t)}=0.$$

Then the differential equation (1) has $P_w(\lambda_0)$ solution, which allows asymptotic representation as $t\to w$

$$y(t)=(\pm1+o(1))e^{\nu\big|(1-\sigma)\,\frac{(\lambda_0-1)^2}{\lambda_0}I_A(t)\big|^{\frac{1}{1-\sigma}}}\,,$$

$$y'(t)=\frac{(2\lambda_0-1)}{(\lambda_0-1)\pi_w(t)}\,\big(\pm1+o(1)\big)\,e^{\nu\big|(1-\sigma)\,\frac{(\lambda_0-1)^2}{\lambda_0}\,I_A(t)\big|^{\frac{1}{1-\sigma}}}\,,$$

$$y''(t)=\frac{\lambda_0(2\lambda_0-1)}{(\lambda_0-1)^2\pi_w^2(t)}\,\big(-1\pm o(1)\big)\,e^{\nu\big|(1-\sigma)\,\frac{(\lambda_0-1)^2}{\lambda_0}\,I_A(t)\big|^{\frac{1}{1-\sigma}}}\,.$$

Here is a consequence of this theorem, if $\sigma=0$, i.e. for the linear differential equation

$$y'''=\alpha_0p(t)y, \tag{3}$$

where $\alpha_0\in\{-1;1\}$, $\sigma\in\mathbb{R}$, $p:[a,w)\to(0,+\infty)$ is a continuous function; $a<w\le+\infty$.

Corollary. *Let for the differential equation (3),*

$$\lim_{t\to\omega}p(t)\pi_\omega^3(t)=c_0>0\ \ \textit{and}\ \ \int\limits_a^\omega\bigg|\frac{p(t)\pi_\omega^3(t)-c_0}{\pi_\omega(t)}\bigg|\,dt<+\infty.$$

Then, if

$$-\frac{16}{36}<\frac{c_0}{\alpha_0}<\frac{1}{3}$$

and

$$\bigg(32\Big(\frac{\alpha_0}{c_0}\Big)^3+36\Big(\frac{\alpha_0}{c_0}\Big)^2-2\,\frac{\alpha_0}{c_0}+6\bigg)^2-\bigg(32\Big(\frac{\alpha_0}{c_0}\Big)^3-2\Big(\frac{\alpha_0}{c_0}\Big)^2+24\,\frac{\alpha_0}{c_0}\bigg)^2\Big(1+\frac{36c_0}{16\alpha_0}\Big)<0,$$

the differential equation (3) has a fundamental system of solutions y_i $(i=1,2,3)$, admitting asymptotic representation as $t\to\omega$

$$y_i(t)=(1+o(1))e^{\big[\alpha_0\,\frac{(\lambda_i-1)^2}{\lambda_i}\,I_A(t)\big]},$$

$$y'_i(t)=\frac{(2\lambda_i-1)}{(\lambda_i-1)\pi_w(t)}\,(1+o(1))e^{\big[\alpha_i\,\frac{(\lambda_i-1)^2}{\lambda_i}\,I_A(t)\big]},$$

$$y''_i(t)=\frac{\lambda_i(2\lambda_i-1)}{(\lambda_i-1)^2\pi_w^2(t)}\,(1+o(1))e^{\big[\alpha_0\,\frac{(\lambda_i-1)^2}{\lambda_i}\,I_A(t)\big]},$$

where λ_i $(i=1,2,3)$ – the roots of the algebraic equation

$$\lambda^3-\lambda^2\Big(3+2\,\frac{\alpha_0}{c_0}\Big)+\lambda\Big(3+\frac{\alpha_0}{c_0}\Big)-1=0.$$

The obtained asymptotics are consistent with the already known results for linear differential equations.

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