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Agent-dynamic modeling of economic systems

Monograph

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The monograph provides the most important results of the research carried out in the ONEU Economic Cybernetics department (2001-2015) within the research theme "Agent-dynamic modeling of economic processes". The basics of the agent-dynamic approach, developed in the Economic Cybernetics department, as well as its selected applications to real economic systems are presented.

Keywords: Agent Based Modeling, Socio-economic Systems, Technological models, Human Resources, Educational Process, Open Economics, Economic Mentality, Sustainability, Ukrainian Economy, Pension Fund.

CONTENTS

Introduction

Multi-agent simulation method in recent decades finds more and more applications in various fields of research [1-3]. The most promising field in the agent-based modeling (ABM) is application of so-called hybrid models in which agent communities act on a background of environment, simulated by non-agent means [2]. Modern systems of computer simulation [3] support ABM in combination with system dynamics and other approaches, which gives the opportunity to develop efficient hybrid models of complex dynamic systems, including economic structures.

This monograph is devoted to one of the new approaches that emerge within so-called hybrid model architectures, which was named “agent-dynamic modeling” (ADM) and discusses its basic principles and selected applications.

The basic idea of ADM is combining the ABM with classical economic balance models, such as von Neumann’s model of expanding economy [4]. ADM allows building hybrid models, making it possible to preserve advantages, and at the same time eliminate the disadvantages of the known classical economic models [5]. As a consequence, ADM can be applied in solution of a number of real economic problems.

The first Section includes an overview of the classical von Neumann’s linear technological model (LTM), as well as the basic ideas of multi-agent approach to modeling, and its implementation in the modern computer simulation systems. The basic ideas of the agent-dynamic modeling approach [5], its analogy with the well-known in physics molecular dynamic simulation approach, as well as the results of its application to real economic systems and its implementation in AnyLogic environment are also presented here.

The second Section Section is devoted to development and improvement of LTM as a basic one within the ADM. Known restrictions and disadvantages of LTM, as well as the problem of their elimination by its so-called "extension" are discussed here. Extended technological models (ETM), in contrast to the LTM, explicitly take into account, along with the production processes, producing and consuming material resources, processes which "produce" and consume labor forces as one of the most important resource of economy.

In the third Section a non-linear version of the extended technological economic model, which takes into account such aspects of the real economy, such as limitations in manpower and production facilities, depreciation of fixed assets and the need to involve investments to promote the economic growth is outlined. ADM makes it possible to eliminate the constraints and synthesize the advantages of such well-known economic models as of von Neumann's LTM, Solow and Samuelson-Hicks. Such non-linear extended technological economic model (NTM) provides a basis for further development of the agent-dynamic approach.

Within the framework of the simplest two-sector version of this model, we consider below some of the possible aspects of modeling of various economic dynamics regimes. The results presented, in particular, occurrence of critical phenomena within NTM, confirm the possibility of its application to the real economies. Some results of the further development of ETM and its use in solution of some crucial problems of the real Ukrainian economy are presented here as well.

The fourth Section deals with issues related to the development of agent-dynamic models of socio-economic systems. Here we demonstrate how ADM can be applied in the study of a number of socio-economic problems. Some examples, which illustrate such the opportunities, are provided. The role of economic mentality factors in the economic and social dynamics is demonstrated by agent-based modeling.

1. From classical economic models to modern simulation systems

The problem of mathematical modeling of complex processes or systems in all fields of science has always been related to the choice of a scale (or level of detailization in their description). The proper scale of the process or phenomenon description depends on the objectives of the study and is usually reflected by prefixes as “micro”, “meso” or “macro”. The first corresponds to the most detailed, and the last, on the contrary, to the most aggregated description.

In molecular physics, for example, which has gained the greatest experience in applying of computer based modeling, micro-level corresponds to use of so-called atomistic models, reflecting the detail of interaction between the smallest objects, preserving the properties of the substances studied, *i.e.* atoms or molecules. The macro-level here corresponds to the model that describes the behavior of matter in general, without explicit consideration of its molecular structure. At the same time the central problem in molecular physics has been and remains the explanation of the observed behavior of macroscopic bodies on the basis of known laws of motion and interaction of their constituent microparticles. For solving this problem usually a well-developed methods of statistical physics are applied [6].

Like macroscopic bodies in statistical physics, macroeconomic objects can not be considered as isolatied, they typically constitute a certain subsystem of a more general system, *i.e.* are usually a part of the open economic systems. At the same time, every open economic system can always be presented as part of the world economy, which certainly is the overall closed economic system. For open economies representing relatively small countries, whose contribution to the world economy is not critical, the world economy serves as a certain "thermostat", providing most of so-called "exogenous" parameters, important for these economies.

Mathematical economics as a scientific field actually emerge only after appearance of the famous John von Neumann's paper "A Model of General Economic Equilibrium" [4], in which the a closed economy is considered very schematically as a cyclically operating set of processes, which consume and produce a finite set of goods. Because at early stages of development of ADM this model served as a prototype of the economic environment in which economic agents operate, below we consider this model in more detail.

1.1. Von Neumann's linear technological model of the economy

Classical von Neumann's linear technological model (LTM) describes the dynamics of *m* processes that produce (and / or consume) *n* goods in discrete time (t = 0,1,2,..):

$$\sum_{j=1}^{n} a_{ij}\, \mathrm{y}_j(t+1) \le \sum_{j=1}^{n} b_{ij}\, \mathrm{y}_j(t), \quad \left(i=\overline{1,n}\right); \tag{1.1}$$

$$\sum_{i=1}^{n} p_i(t+1)\, a_{ij} \le \sum_{i=1}^{n} p_i(t)\, a_{ij}, \quad \left(j=\overline{1,m}\right). \tag{1.2}$$

Here $\mathrm{y}_i(t+1)$ - elements of the column vector of so-called "intensities of the process operation", and $p_i(t+1)$ - are row vectors of the goods prices corresponding to the next period t+1, and related to known values of the same elements $\mathrm{y}_i(t)$ and $p_i(t)$ corresponding to the current period t, and matrix elements a_{ij} and b_{ij} (i = 1, 2…, n, j = 1, 2…, m) are equal to consumption and production of i-th good by j-th process at its unit intensity of operation.

Conditions (1.1) and (1.2) express the requirement that each good produced in the current period, is completely or partially consumed in the next period, and the expences each process in the current period are fully or partially reimbursed in the next period.

If the economy is dynamically balanced, all goods produced in a given period are consumed in the next one. Using contemporary matrix formulation [7] such dynamic material balance can be expressed by the following equation:

$$\mathbf{p}(t)\mathbf{A}\,\mathbf{x}(t+1)=\mathbf{p}(t)\mathbf{B}\mathbf{x}(t)\,. \tag{1.3}$$

Within the framework of the LTM this condition of material balance can be treated a "first law of conservation".

LTM assumes that each good has a common price for all processes. In a balanced economy this leads to another fundamental "law of conservation" – condition of financial balance:

$$\mathbf{p}(t+1)\mathbf{B}\,\mathbf{x}(t)=\mathbf{p}(t)\mathbf{A}\mathbf{x}(t)\,. \tag{1.4}$$

Here $\mathbf{x} = \{x_1,x_2,\dots,x_m\}^T$ – is the column vector of intensity of all processes, and $\mathbf{p}=\{p_1,p_2,\dots,p_n\}$ is the row vector of nonnegative prices of produced goods, t=0,1,… is the index of period (parameter of discrete time), **A** and **B** are (n x m) input and output matrices. Elements a_{ij} are the numbers of good j units required in i-th process operating with its unit intensity, and b_{ij} are the numbers of j-th good units produced by the i-th process operating with the same unit intensity.

The meaning of the dynamic balance equations (1.3) and (1.4) is simple[1]. Equation (1.3) represents the material balance condition: "Everything that produced in the previous period must be consumed during the following period", and equation (1.4) is the financial balance condition: "All expences of the previous period must be reimbursed during the following period".

On the basis of this model, von Neumann showed the possibility of the existence of a dynamic equilibrium in which the growth rates of the balanced economics are related to the interest rates [4].

Von Neumann’s model has been widely discussed in literature, and a review of this discussion can be found in the monograph of Intrilligator [7]. The

[1] It corresponds to a simple model of a fair, where a farmers sell at current prices everything that they had been produced during the year, and buy everything that is necessary for them for the whole next year.

general model proposed by von Neumann combines the features of a microeconomic one (individual processes are considered as elements of the system) and of the macroeconomic approach (being applied to a closed economic system).

Von Neumann proved that there exists a unique solution of the system of equations (and inequalities) of the model which meets the condition of balanced growth of the economy oriented on its maximal efficiency. However, von Neumann's model is difficult to use directly in practical applications not only because of the huge number of factors (intensities of processes and prices of goods) as well as due to its orientation to the a certain type of economy - a planned one, that reflects the well-known Marxist approach in the economic theory. However, his model allows many other solutions for the intensity of the process and goods prices that are compatible with its balance equations. This is a fairly common model of a closed balanced economy, which does not contain any explicit assumptions about the nature of the processes and their driving forces (the mode of production and exchange, motivations and preferences, determining the behavior of individual economic agents).

The balance equations (1.3) and (1.4) play a role analogous to the phenomenological laws of conservation of energy and momentum in physics. They do not allow calculating the trajectory (or intensity) of a single process and the dynamics of the price for individual good, but they provide a common framework for analyzing the behavior of the intensities and prices. Therefore, the balance version of the LTM is not in fact microdynamic, *i.e.* it does not provide equations of the dynamics of individual processes as well as principles of the formation of prices for certain goods.

Although latters may not be uniquely identified from the balance equations (1.3) - (1.4), the von Neumann's approach, as a whole, can serve as a basis for the construction of many other modifications of LTM. Despite the great deal of attention was paid to the development and generalization of LTM [8], this model retains a number of possibilities of extension and generalisation of its elements. Within LTM there remainsome issues, which are still poorly understood, in particular, it is not clear how to take into account the following features:

- Labor force as a specific good;
- Description of an open economy within LTM;
- Limited capacities of the production processes,
- Depreciation of fixed assets;
- Depiction of the a microeconomic dynamics;
- Need in investments for expanded reproduction *etc.*

In the next Sections we discuss some of these issues.

1.2. Modern computer simulation systems for economic studies

Contemporary information technologies of computer simulation in economics are presented by the following main techniques:

- Discrete-event simulation: models the operation of a system as a discrete sequence of events in time (Extend, Arena, ProModel, Witness, Taylor, GPSS/H-Proof etc.);
- Modeling based on system dynamics: focused on the modeling of complex dynamical systems in continuous time (Powersim, Vensim, Dynamo, Stella, Ithink *etc*.);
- Multi-agent modeling (SWARM, RePast, AScape, AnyLogic *etc*.) [11-16].

Nowadays, the dominant trend is the interpenetration of all types of simulation, is symbiosis of various information technologies in modeling, especially for complex applications and complex projects of modeling. Simulation systems have access to procedural languages, making it easy to perform calculations related to planning of factorial simulations, automated optimization and others. An advanced example of such a system is AnyLogic (XJ Technologies St.-Petersburg) [17]. This is one of the few Russian developments in simulation, which has been widely recognized abroad.

AnyLogic supports on a single platform all above approaches: agent-based as well as both discrete-event and continuous modeling (flowchart processes, system dynamics, maps of states *etc*.). AnyLogic [17] is a popular simulation tool, which has more than 15,000 users in 60 countries. This software is intended for the design and optimization of business processes or complex systems, such as production departments, airports, hospitals, *etc*. All methods are implemented in AnyLogic within a single environment and programming language. The focus in good development is made on its flexibility and ease of use for inexperienced users in creating models. Many large companies such as Beeline, Gazprom, General Motors, Mitsubishi, McDonalds chose AnyLogic.

Technological capabilities of advanced simulation systems are characterized by:

- versatility and flexibility, the basic concept of structuring and formalization simulated dynamic processes;
- use of object-oriented specialized programming languages that support the author's design and procedures of the process simulation;
- availability of convenient graphic interface and easy determination of parameters of the model;
- possibility to implement multiple levels of representation of the model. Modern modeling systems use structural and functional approach, multi-level hierarchical nested structure and other ways of presenting models at different levels of description;
- availability of tools for analysis of the results of simulations and of the scenario and variant calculations using models;

- support of mathematical and information analysis procedures, input sensitivity analysis and broad class of computational procedures related to the planning, organization and performing of directed computer simulations;
- integration with other software environments or use their own specialized blocks of analytical and system modeling approach for the implementation of Simulation Data Base, based on access to modeling databases;
- use of execution units outside the modeling system IDE;
- multiplayer mode, distributed interactive simulations, development of interaction in the field of simulation modeling.

AnyLogic environment has a rich set of graphical development tools, which greatly accelerates the process of creating models. AnyLogic is written in the Java programming language, and uses the object-oriented approach. Built-in libraries facilitate the creation of models, besides Java AnyLogic tool platform provides unlimited scalability of models by programming in Java, allow create custom libraries and work with databases. In addition, AnyLogic is a multi-platform software package, supporting the development of models working in a variety of operating systems (Windows, Mac OS and Linux).

AnyLogic allows to use three main approaches to creating simulations of economic models: system dynamics (SDM), discrete event modeling (DEM) and agent-based modeling (ABM).

SDM and DEM consider the system modeled top-down, working on the so-called system level. ABM, on the contrary, uses a bottom-up approach: model maker focuses on the behavior of individual objects. SDM requires a high level of abstraction and is used mainly for strategic level problems. Process-oriented DEM approach is used mainly on the operational and tactical level. The range of applications of agent-based models includes problems of any level of abstraction: an agent can represent a company on the market, a customer, a project, an idea, a vehicle, a pedestrian, a robot, *etc*.

System-dynamic model consists of a set of abstract elements, which represent some of the properties of the simulated system. There are the following basic types of such elements:

- ***Levels*** characterize the accumulated values of the system. This may be goods in stock, goods in transit, bank cash, production facilities, number of employees *etc*. Levels are applicable not only to the physical quantities. For example, the level of awareness is essential when making a decision. Such levels as satisfaction, optimism, and negative expectations affect economic behavior.
- ***Flows*** – are the change rates of levels. For example, the flow of materials, orders, cash, labor, equipment, information.
- ***Decision Functions*** *(valves)* – are functions depending on flow levels. Decision functions may take the form of a simple equation that determines the flow of the reaction to the state of one or two levels. For example, the performance of the transport system can be expressed as the number of goods in transit (a level) and constant (delay during transport). A more complicated example: a decision about hiring workers may be related to the levels of the existing workforce, the average rate of incoming orders, the number of employees undergoing training, the number of newly hired employees, payable on outstanding orders, inventory levels, availability of equipment and materials.
- *The* ***channels of information*** – are elements which connect levels to valves.
- *The* **delay lines** (*lags*) - are used to simulate the delay flows.

Within the DES the performance of the system is represented as a chronological sequence of events. The event takes place at a certain time and marks a change in the state of the system.

In addition to the variables that determine the state of the system and the logic, which determines what happens in response to an event, DEM simulation includes the following components:

- ***Clock*** is the main component of the system, synchronizing changes in the system, *i.e.* the sequence of events occurring in it.
- ***List of events***. Any DEM supports at least one list of events during simulation. *Single-thread systems* are based on instantaneous events that have only one instant event. While DEM simulation of *multithreaded* systems and modeling of such systems support *interval events*, which may have multiple events happening simultaneously. In both cases there are serious problems with synchronization between current events.
- ***Random Number Generators***. DEM systems are divided into the *deterministic* and *stochastic*, depending on how the events are generated, and the main characteristics of *queues*: the time of occurrence of events, the service time, and the number of customers entering the queue per time unit.
- ***Statistics*** *i.e.* basic data, which are collected during simulation in the DEM systems:

  1. Average employment (availability of resources);

  2. The average number of customers in the queue;

  3. The average waiting time *etc.*
- ***The completion condition*** is the occurrence of a predetermined event (*e.g.* achieving a 10-minute waiting time in the queue), or the passage of a predetermined number of the clock cycles of the simulation system.

The main fields of the agent-based modeling are presented in Table 1.1.

Table 1.1.

Application areas of the agent-based modeling

Business and management: • Manufacturing • The market of consumer demand • Logistics • Insurance	**Society:** • Modelling of historical processes • Modeling of deviant behavior
Economics: •Artificial Intelligence • Networks	**Terrorism:** • Social reasons • Organizational networks
Infrastructure: • Electricity Market • The transport industry (including traffic jams and other traffic problems)	**Military developments:** • Optimal management • Scenario planning
Human behavior: • The behavior of the crowd • Modeling of behavior in emergency situations	**Biology:** • Ecology • The behavior of animals in the group • Intracellular interaction of molecules

AnyLogic has several advantages, when compared to other ABM systems: its graphical development environment greatly accelerates the process of creating models; creation of libraries allows developers to re-use the pre-written modules; intuitive graphical user interface simplifies migration from other tools in the simulation by AnyLogic; system is written in the Java programming language, so it is a multi-platform system. Thus, this system can be used for a wide range of problems, including the problems of socio-economic processes modeling.

As an example of such modeling in AnyLogic, one can consider a simplified version of the model of the city development developed by two companies: XJ Technologies and the ATN, in the framework of the project carried out for the municipal government of one of the cities of France. The model is based on agents, *i.e.* on certain independent objects of several fixed types, each with its own behavior characteristics and parameters. The model allows predicting the traffic load in the city, degradation of environment due to gas emissions and resettlement of families with the areas of the city. It reflects the labor market and housing market, as well as the availability of space for production deployment. The model can reveal, for example, the following effects: when the population increases the jobs availability is reducing and affordable housing is also reduced and housing problems leads to an increase in rents, which in turn will lead to greater congestion, unemployment and low wages [18, P.327].

This model is a demonstration. It shows the possibility of taking into account the influence of some factors and characteristics in a complex system on others using the paradigm of agent-based modeling approach. For example, using this model one can trace how the investments in new housing construction or improvement of conditions for opening new businesses affect the dynamics of the development of the city, in particular, transport problems and pollution. In addition, within this model one can change the number of houses (apartments) and businesses as well as level of comfort in each area and see how the system develops due to these changes [18, p.331].

It can be concluded that the existing software implementing the ABM method, offer enough diverse tools allowing solving fairly complex problems by modeling both economic and social phenomena.

1.3. Agent-based tools for modeling of socio-economic processes

Due to by rapidly advancing information technologies and the acceleration of globalization, there is a need for a more visual modeling techniques for more effective enterprises management, business process management and social problems. Significant distribution received such a modeling method as Agent Based Modeling (ABM).

ABM is a relatively new (1990-2000) trend in computer simulation, which is used to study the decentralized systems, the dynamics of the operation of which is determined by activity of individual elements of these systems. Thus, the agent-based models comprise a number of interacting active objects that reflect objects and relationships of the real world. Individual behavior of each agent forms a global behavior of the simulated system. Main aim of the agent-based models is to get an idea about the general behavior of the system based on the assumptions of the individual, the private behavior of its individual active objects and about the interaction of these objects in the system. Each agent is completely autonomous and independent: it makes targeted actions, comes into contact with other agents, is able to make decisions, use resources available to him, to move, to adapt to the environment (evolve), *etc*.

ABM is a set interacting active objects (agents) which reflect the objects and relationships in the real world. Each agent is completely autonomous and independent: it makes targeted actions, comes into contact with other agents, is able to make decisions to move in and evolve [5].

Multi-agent models are used to study the decentralized systems, not determined by the global rules and laws, but on the contrary, these global rules and laws are the result of the activity of individual team members. The goal of ABM is to get an idea about these global rules, the general behavior of the system based on the assumptions of the individual, the private behavior of its individual active objects and the interaction of these objects in the system.

Thus, agent in ABM can be defined as an entity possessing 1) activity, 2) autonomous behavior, 3) it can take a decision in accordance with a certain set of rules, 4) it can interact with the environment, as well as 5) it can change itself independently [3].

During last decates modeling of social processes attracts lot of attention of scientists. ABM method here is the most convenient because it allows you to take into account the behavior of individual actors (agents) of the social system. Multi-agent models are very sutable just for investigation of decentralized socio-economic systems whose global rules and laws arise from the activity of individuals.

Development of software tools that implement ABM method makes is possible to describe the behavior and interaction of socio-economic agents with little or no simplifications. The importance of ABM in its practical computer implementation in modeling of socio-economic processes can hardly be overestimated.

ABM methods allow to study the behavior and to establish the general laws of behavior of social and economic systems by specifying particular rules of behavior (psychology) of economic agents, participants of labor relations. Multi-agent simulation has become a convenient tool for investigating the impact of these changes and the impact of restructuring processes of consciousness agents to their behavior in the real world [4].

Simulate a realistic situation in such a complicated field is usually rather difficult, because one needs to consider many factors and sometimes the results are not completely and adequately reflect the reality. Within the framework, for example, of system-dynamic model it is difficult to track all the details of the interaction of multiple market segments, but with an agent based model it is quite simple and intuitive. This makes it possible to keep track of much more information about the system studied.

Agents in ABM may represent not only people, but organizations, enterprises, even whole countries. Agents act as independent purposeful entities with identifiable skills and qualities that are subject to change due to the interaction with other agents and environment. Their behavior is determined by some adaptive set of rules; they may demonstrate a variety of modeling behavior. Agent-based models allow one to create persuasive visualizations, based on the behavior of agents in a variety of scenarios. They are successfully used in modeling such complex processes as crises, giving practical solutions of various problems.

Using ABM enables, in particular, quite accurately reflect the nature of the real interaction between people working together, which depends on their mentality. The idea of such an interaction is essential for decision-making in the introduction of innovative technology solutions. Managers often say that the most valuable assets of the company - those people who come to work and go home every day. However, no other method of enterprise-level simulation gives the characteristics of the independent behavior of employees or their interaction with each other better than ABM. It also allows considering the probabilistic behavior of agents, which reflects the indeterminacy of the real world.

For example, in the model of agents-researchers one can include the decision rules establishing the likelihood of communication between individual researchers and the exchange of complementary knowledge necessary for development of the good. Unlike ABM, other methods, such as business process modeling, are deterministic and do not take into account the motivation of employees and the variability of their behavior.

Interest to ABM in the study of emergent properties of complex adaptive systems is constantly growing. Currently, the market offers at least 50 different tools using ABM.

Below we discuss several software goods that employ agent-based approach and are useful in computer simulation of social and economic processes.

One of the actual problems can not be solved by other means, is a simulation of population segregation on racial, linguistic, national characteristics. Thomas Schelling, the 2005 Economics Nobel Prize winner, built an agent-based model called the "Schelling segregation model" for explaining why phenomena that people are intolerant, and don't want to live next to people that are different from them.

The agents of this model are individuals presented by pieces of three different colors, arranged in any order on the square area (8 by 8), surrounded by the same individuals as they are. Each individual agent may be a "happy" or "unhappy", depending on the number of its surrounding neighbors agents having the same color [11,12]. When one runs this model, the agent (a person) can move over a conditional city (64 cells), and even leave it. The program allows defining "the conditions of a happy co-existence" of an individual for each of the three groups. For example, a green color agent "happy" if at least three of the eight neighbors have the same color with it. After reaching a predetermined neighborhood, the agent becomes "happy" and stops at the choice of the square as a "place of residence".

Depending on the settings of the program, the "poor residents" can either leave the city, or select the next vacant space. Model of segregation may be used when modeling urban social stratification and racial (urbanization problem), and the object of modeling can be a city, and, for example, a division of the industrial enterprise.

NetLogo [12] is a programmable environment designed to simulate the natural and social phenomena. The goodwas developed in 1999 and its support and upgrading is performed by the Center for Connected Learning and Computer-Based Modeling relocated to Northwestern University, Illinois, USA in 2000.

NetLogo language is an extension of Logo programming language and supports publishing online java-applets that can be used for modeling.

The site NetLogo [12] presents a library of programs using agent-based approach and implemented in the form of Java-applets, launching directly in the browser. In one of these models, called ‘Altruism’, there are two types of agents: altruistic and selfish. A basic premise of the model lies in the fact that these two types are fighting for the surrounding area, with altruism and selfishness of some others is the most important genetic trait.

Initially all agents are settled in random order on a certain territory. The user sets their own percentage of the territory occupied by each of the warring parties. The cost of the site occupied by the selfish agent is above the cost of the site, which settled an altruist. For his altruism agent has to "pay" and the value of its area decreases. There is a struggle for the empty land. Vacant land resembles a field sown with seeds of two types. If the dominant in the area are crops of altruists, there appears a new agent - an altruist, otherwise he becomes an egoist.

Victory in the struggle depends on the value of the neighboring areas as land occupied by selfish, have an advantage in price. It is easy to imagine that, in the end, the selfish population prevails. However, the program takes into account two other factors - resistance to the area colonization and the probability of death at settling. The simulation results vary as soon as environmental conditions become more hostile to the existence of agents. In this case, altruists not disappear, but dominate over the selfish.

The "Prisoner's dilemma" model [13] has received its name from the gaming scenario in which two criminals, arrested for a minor offense, are interrogated separately. Each of them has the possibility to "keep silent" (cooperate with his accomplice), or go to the police side by denunciation in exchange for some concessions. On the basis of the decision, the criminals get points. If both remain silent, then each gets 3 points (the police has no evidence against them and could incriminate only a minor offense). If one is silent, and the other "works" with the police, the "traitor" gets 5 points, and his silent accomplice no points. If the police agree to "work" the two, then they get only one point. The biggest win of 6 points is for "silence" of both criminals. The gain of 5 points is obtained if one of the criminals is "silent" and the other one agrees to denunciation. Mutual betrayal of two criminals gives them the win by 2 points. Thus, the best option for the team system is collaboration. But personally for each of the criminals 'work' with the police is a chance to earn more points. Indeed, if your partner is "silent", you get only 3 points, if you are "silent" too and the whole 5 points if you agree to "work" with the police. The second option, your partner "works" with the police, then you do not get any points, if are "silent", and one point if you agree to "work" with the police. An important conclusion is: the irrational (in economic terms) behavior of each individual can bring the best result to the whole team.

Trade Network Game Laboratory (TNG Lab) [14] develops software goods for the study of the formation and development of trade networks among strategically interacting agents (buyers, sellers and dealers), working on different market strategies. TNG Lab uses an extended theory of conformity and combines it with the theory of dynamic games in which actors should determine the choice of trading partner and cooperation strategy.

The StarLogo TNG laboratory software released in 2010 allows mastering quickly the basic techniques of ABM, understanding independently particular working models and learning how to create own models. The opportunities provided by the program are easy to understand by the "epidemic" model example.

Within this model the living entities (agents) inhabit a certain territory. With the help of the options menu, user can specify the initial location of all agents, the pace and trajectory of their movement on "the territory of residence". Initially, all agents are of the same color (green).

Each agent has a probability to be infected. Following the simulation of infection some small part of the agents of changes their color to red (become sick). Moving in space, infected agents spread the infection. The collision of a healthy and infected agent is condition of the infection, then the healthy agent turn red. In addition, part of the creatures immune to infection and therefore do not become infected when confronted with infectied. Having recovered agent becomes healthy again. User-friendly interface allows you to monitor in graphical form of spread (decay) of epidemic. The program is easy to use and allows you to quickly create a simple agent-based model.

The name of ABM «Repast» system is an acronym for «Recursive Porous Agent Simulation Toolkit» [15]. The package was originally developed by the University of Chicago. Repast has several versions in different languages, as well as built-in adaptive features, such as genetic algorithms and regression. The tool also includes other methods of modeling (system dynamics) and can be used for modeling of social networks. In addition, Repast includes support of geographic information systems for geographic modeling purposes.

1.4. Agent-dynamic approach in economic modeling

In economic theory, two approaches are used for analysis and modeling of systems. The microeconomic approach is based on a detailed study of the minimal element of the economic system, which has all the necessary features of the economic entity (*e.g.* enterprise). Macro-economic approach, on the contrary, studies the processes in the economy described by averages, aggregates, characterized by a set of interacting economic elements constituting, for example, national or global economy.

Agent-dynamic simulation actually links these two approaches, combining the economic dynamics at the macro-level and micro-economic dynamics of individual processes and market dynamics of prices for individual goods.

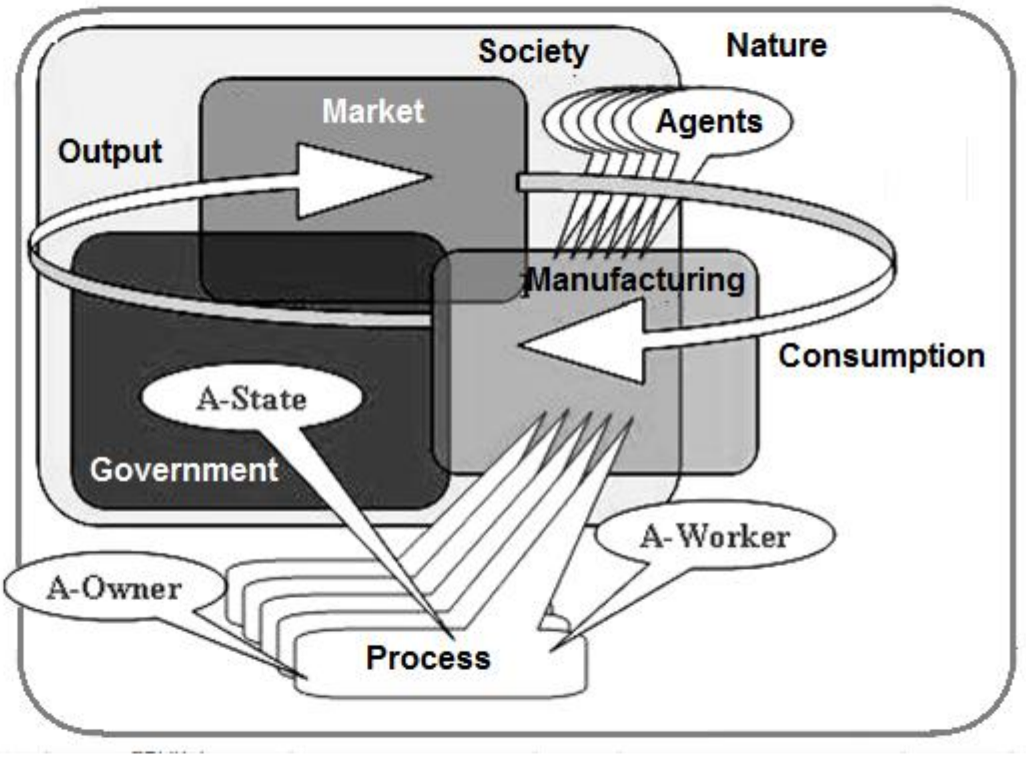

Fig.1.1. The architecture of the general agent-dynamic model of economy.

For this purpose the system of LTM equations (1.3) - (1.4) is replaced by a somewhat more general, writable separately for each process and each good (as it described below in Section 2.3). At the same time, any solution of this generalized system still satisfies the balance equations (1.3) - (1.4).

Economics within ADM approach is considered as a system which includes a number of production processes, which evolve in dynamic environment and interact with a set of agents. Various types of agents, interacting with each other and with the dynamic production processes are introduced. This system can be considered even more generally, as immersed into nature as presented in Fig. 1.1. The general scheme of ADM economy shown there reflects the correspondence between dynamic processes and at least three types of agents (A-Owner, A-Worker and A-State). A-Owner agents represent owners and managers governing the production processes (companies, enterprises *etc*.), A-Worker agents are linked with these processes as all A-Owners and A-Workers are acting in the legislative field, which is determined by the government (A-State agent).

Below we discuss new opportunities offered in the ABM when using this ADM hybrid model architecture. The basics of the agent-dynamic modeling [5,18] are explained below and several examples of its implementation within AnyLogic environment [19] are provided.

When implementing the agent-dynamic approach, like any other hybrid model, it is very important not only to realize the benefits of ABM, but also to apply the adequate model of non-agent-based environment in which agents operate. The following chapters are devoted to modeling of such non-agent-based model environments, which can be used within ADM.

2. Extended technological model as a basis for the agent-dynamic approach

2.1. Labor force in technological model

Necessity of inclusion of labor force (manpower or human resourses) to technological model of economy was noted in papers of many economists. But most often, human resourse (HR) is not considered as a factor reproducible within economics but accounted as a separate element, which is not related to other economic factors, but only satisfies certain inequalities (for example, the total number of labor force used by all processes, can not exceed the number of the working population). In other works labor force was included directly into the costs matrix so that the number of the costs vector elements is incremented by one.

Such interpretations do not reflect the specificity of the origin of HR, which in reality are always limited and require certain costs for their reproduction. In this formulation, this problem was considered in the framework of LTM in [20].

HR is considered as a "good" having the following specific features:

1) It is consumed by all production processes;

2) It is not reproduced by any of the production processes;

3) It is limited and indestructible, *i.e.* it may be limiting the economy (full employment) or be surplus (unemployment).

In addition, HR, as well as some other goods, can not be produced or consumed in one production cycle. In this sense, HR like capital assets as well as durable goods are is formally "consumed" within many technological periods, and their consumption can be seen as a kind of depreciation. At the same time the labor force is actually produced by a specific non-production process, which is a natural process of social development.

$$\hat{\mathbf{A}} = \begin{array}{cc} (0 \leftarrow \textit{processes} \rightarrow m) & \\ \begin{pmatrix} a_{00} & a_{01} & \cdots & a_{0m} \\ a_{10} & a_{11} & \cdots & a_{1m} \\ a_{20} & a_{21} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n0} & a_{n0} & \cdots & a_{nm} \end{pmatrix} & \textit{products} \end{array} \qquad \hat{\mathbf{B}} = \begin{array}{cc} (0 \leftarrow \textit{processes} \rightarrow m) & \\ \begin{pmatrix} b_{00} & 0 & \cdots & 0 \\ 0 & b_{11} & \cdots & b_{1m} \\ 0 & b_{21} & \cdots & b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & b_{n0} & \cdots & b_{nm} \end{pmatrix} & \textit{products} \end{array}$$

Fig. 2.1. Extended input $\hat{\mathbf{A}}$ and output $\hat{\mathbf{B}}$ matrices.

An extension of the von Neumann’s technological model, wich explicitly takes into account an additional non-industrial (public) process, which allows reproducing of labor resources was proposed in [20].

The general formulation of the problem is as follows. It is necessary to complement the von Neumann's model by HR as a particular “goods ” which is consumed by all, without exception, production processes, but reproduced only by one (social, natural) process in non-productive sphere. It includes also education and medicine, and considered an integral part of that "public" process, which recreates HR.

In the extended vectors of intensities $\hat{\mathbf{y}}(t)$ and prices $\hat{\mathbf{p}}(t)$ there are additional elements x_0 and y_0:

$\hat{\mathbf{y}}(t) = \{y_0, y_1, y_2, \ldots, y_m\}^T$ - is the extended column vector of intensity of processes, and

$\hat{\mathbf{p}}(t) = \{p_0, p_1, p_2, \ldots, p_n\}$ – is the extended row vector of prices.

Matrices **A** and **B** are complemented accordingly by zeroth rows and zeroth columns and form extended (m+1 x n+1) matrices $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ as it shown in Fig.2.1.

Note that HR marked here by the index x_0, are consumed both in production (j = 1, 2,.., m), and in non-productive (j = 0) processes. Elements a_{0k}, (k = 1, 2…, m) of matrix $\hat{\mathbf{A}}$ reflect the the measure of labor resources consumption in the industrial sector, and all $a_{0k} > 0$, because none of the production process cannot function without HR.

The equations of the extended model, after replacement of vectors **y**(t) and **p**(t) by $\hat{\mathbf{y}}(t)$ and $\hat{\mathbf{p}}(t)$, and **A** and **B** matrices by $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$, correspondingly, match formally to equations (1.3) and (1.4). However, consideration of the features of labor resources allows simplifying of these equations.

If we consider the number of people employed in the *k*-th production process for measure of its intensity, the element a_{0k} becomes the number of new HR, involved in the current period to the *k*-th process per one already working in it. A special role is played here by a_{00} element, which means the consumption of HR in the non-productive sphere

A natural measure of the non-productive sphere intensity x_0 is the total number of employable population, which provides a reproduction of labor force (including employed in all industrial sectors, as well as those employed in the field of education, medicine *etc.*, as well as disabled and temporarily unemployed).

Other additional elements of the first line a_{0k}, $k = 1, 2\ldots, m$ of matrix $\hat{\mathbf{A}}$ have the meaning of the specific consumption of all goods produced by the economy in the non-production sector (*i.e.* non-productive consumption, absent in the original LTM).

Importantly, in the extended ($m + 1$ x $n + 1$) output matrix $\hat{\mathbf{B}}$ only one additional element b_{00} reflecting the natural increase of the population (the difference between birth and death rates) is nonzero. All other additional elements with at least one zero index are zero: $b_{0k} = b_{j0} = 0$, ($k = 1,2\ldots,m$; $j = 1,2\ldots,n$), because neither the production process does not produce labor, and non-productive sphere can not produce any other good except the labor force.

An additional element of the extended vector of prices - the price of human recources p_0, is the measure of public expenditure on reproduction of the labor force[2]. Prices of conventional goods are non-negative, but they may be zero. But the price of labor force can not be zero, *i.e.* always $p_0 > 0$, since people always need their livelihoods.

In view of this the extended model the balance equations (1.3) - (1.4) could be written in terms of the original inpu matrix **A**, output matrix **B**, vectors **y** and **p** as follows:

1) Material balance:

$$\mathbf{p}(t)\mathbf{A}\,\mathbf{y}(t+1)+\sum_{j=1}^{n} p_j(t)\,a_{j0}\,y_0(t+1)=\mathbf{p}(t)\mathbf{B}\mathbf{y}(t), \tag{2.1}$$

$$p_0(t)\sum_{k=0}^{m} a_{0k}\,y_k(t+1)=p_0(t)\,b_{00}\,y_0(t). \tag{2.2}$$

2) Financial balance:

$$\mathbf{p}(t+1)\mathbf{B}\,\mathbf{y}(t)=\mathbf{p}(t)\mathbf{A}\mathbf{y}(t)+p_0(t)\sum_{k=1}^{m} a_{0k}\,y_k(t)+\tau\,p_0(t)\,a_{00}\,y_0(t), \tag{2.3}$$

$$p_0(t+1)\,b_{00}\,y_0(t)=\sum_{j=1}^{n} p_j(t)\,a_{j0}\,y_0(t)+(1-\tau)\,p_0(t)\,a_{00}\,y_0(t). \tag{2.4}$$

Parameter $\boldsymbol{\tau}$ $(0 < \boldsymbol{\tau} < 1)$ in equations (2.3) - (2.4) takes into account the different social models of the labor forces reproduction reimbursement.

In the case $\boldsymbol{\tau} = 0$ these costs are completely reimbursed by the society (the liberal model), in the limit $\boldsymbol{\tau} = 1$ these costs are fully covered by the production assets (socialistic model).

This type of extended LTM can be used in modeling of the labor forces dynamics, of the education system, in the planning of the needs in manpower and creating new jobs. Also, using this model, one can investigate the relationship

[2] It reflects all costs, including pensions.

between the cost of production processes and the cost of living, unemployment and predict the dynamics of the state expences needed to support unemployed.

2.2. Extended technological model of an open economy

As it has been noted above, the original von Neumann's model is applicable only to a closed economy. At the same time, most of the national economies are actually open. Every company, corporation or independent state when exchanging goods with others maintains its own financial balances.

To study the open economies in this case it is necessary to use models taking into account import and export. The model of the open economic system of any country provided to be an open system should reflect economic ties with other countries. In this case, we can start from the general model of the only known closed economy – the world economy [22], which encompasses the open economies of all countries. If the open economy considered is a corporation or country making only a small contribution to the world economy, the problem of modeling of such an open economy is considerably simplified, since it is not necessary to consider its inverse effect on the world economic system

Attempts to describe an open system based on a closed technological model can be found in many works related to the study of inter-regional cross-sectoral balances, for example, in [22] and [23]. The idea to extend von Neumann's approach to open systems was proposed in [24]. Its main idea is as follows.

From the whole set of LTM processes of a closed system a subset which relates to a separate independent national economy is selected. Correspondingly, from the complete set of LTM equations only that part which describes that national economy exchanging goods with the rest of the (outer) part system is considered.

In turn, the whole set of n goods, produced in the world economy, is subdivided into two parts: a subset of n_D (j = 1, 2,.., n_D) goods, which are produced in the given country (let's call them, for definiteness 'domestic'), and the rest of n_F =n - n_D goods (j = n_D + 1..,n) produced in other countries considered as "foreign" goods [3]. In general, due to insularity of the world economy n = n_D + n_F.

Vector **y** consists from m_D intensities of "domestic" processes (k = 1, 2,., m_D) related to m-dimensional vector $\boldsymbol{y}_D$ and m_F of the "foreign" processes intensities (k = m_D + 1,., m), defined by m_F - dimensional vector $\mathbf{y}_F$, and due to insularity of the world economy: $m = m_D + m_F$.

Input matrix **A** can be now re-written ib block form as:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_D & \mathbf{A}_E \\ \mathbf{A}_I & \mathbf{A}_F \end{pmatrix}, \qquad (2.5)$$

where the (n_D x m_D) sub-matrix $\mathbf{A}_D$ takes into account consumption of domestic goods by domestic processes (domestic consumption), the (n_D x m_F) sub-matrix $\mathbf{A}_E$ - (foreign consumption of goods produced by domestic processes) accounts for export, the (n_F x m_D) sub-matrix $\mathbf{A}_I$ - (consumption of foreign goods by domestic processes) accounts for import, and (n_F x m_F) sub-matrix $\mathbf{A}_F$ takes into account consumption of foreign goods by foreign processes.

Output matrix **B**, according to the above subdivision of goods looks like this:

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_D & 0 \\ 0 & \mathbf{B}_F \end{pmatrix}, \qquad (2.6)$$

Here the square sub-matrices: $\mathbf{B}_D$ (n_D x n_D) and $\mathbf{B_F}$ (n_F x n_F) characterize, respectively, outputs of domestic and foreign processes[4].

Each good, both domestic and foreign, has a certain market value in each period. Combining the prices of all domestic goods, we obtain n_D - dimensional

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[3] Note that domestic goods that are identical to the 'foreign', should be considered separately because they may have a different market price.
[4] Matrix **B** contains zeros as transnational corporations does not taken into account in this model.

$\mathbf{p}_D$ vector, and vector of prices on foreign goods $\mathbf{p}_F$ has accordingly n_F components.

For of the world economy as a closed system described within the LTM two balances - material (1.3) and financial (1.4) must be fulfilled. To solve the problem of describing an open economy we need a subsystem of equations that describe the domestic economy, interacting with the world one by importing and exporting goods. This can be done, if we note that for each country as well as all other countries must exist the material balance of the foreign trade:

$$\begin{cases} \mathbf{p}_D(t)\,\mathbf{A}_D\,\mathbf{y}_D(t+1)+\mathbf{p}_D(t)\,\mathbf{A}_E\,\mathbf{y}_F(t)=\mathbf{p}_D(t)\,\mathbf{B}_D\,\mathbf{y}_D(t) & (2.7) \\ \mathbf{p}_F(t)\,\mathbf{A}_F\,\mathbf{y}_F(t+1)+\mathbf{p}_F(t)\,\mathbf{A}_I\,\mathbf{y}_D(t)=\mathbf{p}_F(t)\,\mathbf{B}_F\,\mathbf{y}_F(t). & (2.8) \end{cases}$$

It should be noted that in contrast to the internal financial balance, in order to buy some of the goods on the foreign market, each country[5] must have an equivalent amount of foreign currency. This sum it can get only when sells abroad an equivalent amount of other goods and the total revenue from the domestic exports of the period in the foreign currency is spent on the same period import.

Bearing in mind the zero total balance of foreign trade (export-import) of the domestic open economy, we recognize that sum of money spent by individual country on the purchase of imports in a certain period, should be compensated by exports during the same period:

$$\mathbf{p}_D(t)\,\mathbf{A}_E\,\mathbf{y}_F(t)=\mathbf{p}_F(t)\,\mathbf{A}_I\,\mathbf{y}_D(t).$$

Counting this foreign (currency) balance, we can write:

$$\begin{cases} \mathbf{p}_D(t+1)\mathbf{B}_D\,\mathbf{y}_D(t)=\mathbf{p}_D(t)\,\mathbf{A}_D\,\mathbf{y}_D(t)+\mathbf{p}_F(t)\mathbf{A}_I\mathbf{y}_D(t), & (2.9) \\ \mathbf{p}_F(t+1)\mathbf{B}_F\,\mathbf{y}_F(t)=\mathbf{p}_F(t)\,\mathbf{A}_F\,\mathbf{y}_F(t)+\mathbf{p}_D(t)\mathbf{A}_E\mathbf{y}_F(t). & (2.10) \end{cases}$$

Equation (2.9) shows that all expences of a country in the current period (consumption of their own and imported products), must be reimbursed in the next period. Equation (2.10) expresses the same for all the other countries.

[5] Валюта которой свободно не конвертируется.

In the case when the share of a given country with open economy in the world foreign trade is small, the system of equations describing the open economy within LTM can be further simplified by reducing to only two equations [24]:

$$\begin{cases} \mathbf{p}_D(t)\,\mathbf{A}_D\,\mathbf{y}_D(t+1) + \mathbf{p}_D(t)\,\mathbf{A}_E\,\mathbf{y}_F(t) = \mathbf{p}_D(t)\,\mathbf{B}_D\,\mathbf{y}_D(t)\ ; & (2.11) \\ \mathbf{p}_D(t+1)\,\mathbf{B}_D\,\mathbf{y}_D(t) = \mathbf{p}_D(t)\,\mathbf{A}_D\,\mathbf{y}_D(t) + \mathbf{p}_F(t)\mathbf{A}_I\mathbf{y}_D(t). & (2.12) \end{cases}$$

Formally, the equations (2.11) and (2.12) coincide respectively with equations (2.7) and (2.8), but the vectors $\mathbf{y}_F(t)$ and $\mathbf{p}_F(t)$ are playing here a role of exogenous factors, i.e. they could be calculated independently, for example, using the LTM equations for the closed world economy.

Modification of LTM considered above provides the possibility to study links between the economies of individual countries and the global economy and, in particular, to use the economic model described above, extended by incusion of labor force dynamics, to investigate the migration processes dynamics in open economic systems.

Such an approach, proposed in [24], could be developed in the furture by adding to it, for example, duties to the cost of the goods that are imported or exported. Using such model of an open economy, we can also examine the content, size, and structure of exports, imports and production for the domestic market. Since the same good can be considered here as a good of the domestic and foreign economy, it is possible to predict the dynamics of prices, both on domestic and international markets.

Using the equations of an open economy (2.11) - (2.12), one can investigate the degree of dependence of the domestic country on certain imported goods, and analyze the competitiveness of own export goods abroad, *etc*.

In this formulation, the technologicsl model of open economy is important, for example, to simulate the impact of the global economic dynamics on the development of transforming economy of Ukraine. One can apply the above-described model of an open economy, additionally extended by account of labor forces as was shown above. This modification can be useful, for example, for planning needs in the labor force in the economy of Ukraine after the crisis due to the ongoing demographic decline. With this model, one can investigate also the relationship between the cost of production processes and the cost of living, to display the dynamics of unemployment and state expences on maintaining the unemployed, *etc*.

2.3. Description of microeconomic dynamics within LTM

As mentioned above, within the balanced LTM of expanding economy there exist a unique solution only on so called “turnpike”, *i.e.* on the trajectory where the intensities of all processes are changing with a maximum rate, and the prices, respectively, change with the inverse rate [4]. For all other cases, there may be multiple solutions of equations (1.3) - (1.4).

The real economy, for various reasons, never reaches that maximum rate, but the general fundamental balances such as material balance (what is consumed, must first be produced) and financial balance (what has been spent, must first be earned) are generally fulfilled. Although such a simple model has many nuances which are not taken into account (*e.g.* reserves or loans), the introduction of the market-sharing mechanism between processes does not mean the complete abandonment of these fundamental balances.

In this regard, it is interesting to consider a more general problem: how behaves the economy defined by a set of processes and goods, which is not on the turnpike, but the processes intensities and goods prices it which satisfy the balance relationships (1.3) and (1.4)? In this case the financial balances of the individual processes and material balances of individual goods can be violated locally, reflecting the existence of reserves or credits mentioned above and ignored by the model.

To solve this problem it is necessary to specify additional equations that determine the dynamics of intensities of individual processes and prices of individual goods in the economy obeying only global dynamic equilibrium, but alowing for local deviations from it (fluctuations). Such problem statement is similar to the transition in statistical physics from a purely phenomenological approach (for example, from thermodynamics) to the kinetic, what requires usage of additional (kinetic) equations [33].

Unfortunately, the economic theory gives no quantitative microscopic equations of motion analogues to those for individual physical particles. At the same time the main qualitative factors that determine the behavior of the economic agents are known. They are:

1) The pursuit of maximum profit;

2) The desire for expansion on the market *etc*.

This Section provides a dynamic model of a closed economy, which takes into account these microeconomic incentives. On the one hand, this model is formulated in the "market" terms of the desire for profit or expansion in the market, and on the other - it is compatible with macroeconomic balance equation (1.3) - (1.4). This problem was first considered in [25]. Below we provide its formulation and discuss main results.

Let's determine two vectors: vector-increment $\mathbf{\Delta x} = \mathbf{x}(t+1) - \mathbf{x}(t)$ of intensities of all the processes and vector- increment $\mathbf{\Delta y} = \mathbf{y}(t+1) - \mathbf{y}(t)$ of all goods prices in the next period compared to the previous one, without introducing the hypothesis of a local dynamic equilibrium in the system.

The mathematical formulation of this agent-dynamic model in discrete time is defined by the following system of equations for the increments of individual processes intensities (j = 1,.. m):

$$\frac{\Delta y_j}{x_j} = \varepsilon \sum_{i=1}^{n} \delta_{ij} \alpha_i + (1-\varepsilon)\beta_j \ , \tag{2.13}$$

and for increments of prices for all goods (i=1,2,…n):

$$\frac{\Delta p_i}{y_i} = \sigma \sum_{j=1}^{m} \frac{\gamma_{ij}}{\beta_j} + (1-\sigma)\frac{1}{\alpha_i}. \tag{2.14}$$

Here we have introduced the following notation for the variables:

1) rate of change α_i of the i-th good output:

$$\alpha_i = \frac{\sum_{k=1}^{m} b_{ik} x_k}{\sum_{k=1}^{m} \alpha_{ik} x_k} - 1; \tag{2.15}$$

2) profitability factor β_j of the j-th process:

$$\beta_j = \frac{\sum_{k=1}^{n} y_k b_{kj}}{\sum_{k=1}^{n} y_k \alpha_{kj}} - 1; \tag{2.16}$$

3) share of k-th process in the production of i-th good:

$$\gamma_{ik} = \frac{b_{ik} x_k}{\sum_{p=1}^{m} b_{ip} x_p}; \tag{2.17}$$

4) share of j-th process in the consumption of k-th good:

$$\delta_{kj} = \frac{y_k a_{kj}}{\sum_{k=1}^{n} y_k \alpha_{kj}}, \tag{2.18}$$

ε and σ are parameters of the model.

Note that the increments of intensity of processes $\Delta x_j = x_j(t+1) - x_j(t)$ and the price changes $\Delta y_i = y_i(t+1) - y_i(t)$ in the next period compared to the previous one calculated according to the equations ADM (2.13) and (2.14), when substituted into balance equations (1.3) and (1.4) turn them into identities at any ε and σ.

Let us consider the meaning of these parameters. At $\varepsilon \to 0$ the rate of change in the intensity of *j*-th process in equation (2.13) tends to its profitability factor β_j (2.16). And when factor $\varepsilon \to 1$, the rate of change in intensity of *j*-th process tends to the weighted over the share of consumed goods average value of the rate of change of their release. Thus, the parameter ε in equation (2.13), varying within $0 < \varepsilon < 1$, characterizes the economic policy (marketing strategy), that always takes into account two factors: 1) profitability of production and 2) the prospects of the produced goodon the market.

The rate of change of *i*-th good price, according to equation (2.14), is also determined by factors α_i (2.15) and β_j (2.16), but is inversely proportional to each of them.

At $\sigma \to 0$ in equation (2.14), the price of *i*-th good falls at a rate inversely proportional to the rate of change in the release of this good, and when $\sigma \to 1$, the rate of change of prices of *i*-th good is equal to the inverse of their profitability averaged over all involved in its production processes.

Thus, the parameter σ characterizes the overall pricing policy of the whole economy. With increasing σ from zero to unity market pricing is becoming increasingly profit-oriented, and less expansionary.

By direct substitution is easy to prove that the elements of the vectors-increments of processes intensities $\Delta\mathbf{y}$ and prices $\Delta\mathbf{p}$ calculated according to the equations (2.13) and (2.14), in view of definitions (2.16) - (2.18) reduce to identities balance equations (1.3) - (1.4), *i.e.*, they are compatible with them at any values of ε and σ.

Note that rates of change of the various processes intensities are linked by positive feedback. Thus, we can conclude that the parameter ε in equation (2.14) characterizes the *economic development policies* of the economic entity (or its marketing strategy), that always takes into account two factors - profitability and market trends (the market prospects of the produced good). With increasing ε from zero to unity the development policy of production is becoming to a lesser extent profit-oriented and increasingly expansionary (*i.e.*, oriented on manufacturing of perspective goods).

Parameter σ characterizes the *pricing policy* (for example, of a corporation). With increasing σ pricing is becoming more stimulating (focused on the growth of consumption) and less profit-oriented.

Note that in the extremely monopolized economy the share of each process in the production of the good tends to unity: $\gamma_{ij} = \delta_{ij}$, (here δ_{ij}, is the Kronecker delta). At the same time increasing the value ε to unity at zero parameter σ (when both the development policy and pricing policy are oriented only on the growth of output) the system of equations (2.13) - (2.14) splits into two independent subsystems, *i.e.* the rates of changing prices become non-correlated with the growth of output. It should certainly lead to the destabilization of the economy. Therefore, the most urgent task in application of this model is studying of the stability of economic dynamics with an increase in of parameter of the policy ε.

Below, we provide a simple example illustrating the features that appear in the dynamic behavior of economic system and analyze the role of the development policy and pricing policy factors, modeled within the microeconomic modification of LTM (2.13) - (2.14).

The model described the ultimately monopolized economic system consisting of four processes and producing four goods: $n = m = 4$ (diagonal output matrix B). Their developments were simulated using equations of microeconomic dynamics (2.13) - (2.18) (see Section 2.3). The elements of input and output matrices have been defined so that at $\varepsilon = \sigma = 0$ the overall technological rate of the growth was five percent. The initial prices and intensity have been set in accordance with the eigenvector corresponding to the 0.05 Frobenius-Perron eigenvalue of the $\mathbf{A}^{-1}\mathbf{B}$-$\mathbf{E}$ matrix, to get the proper turnpike as an initial state.

We studied the effect of varying the development policy ε and pricing policy σ settings in (2.13) and (2.14) on the behavior of the intensities and prices. Along with the numerical solution of equations (3.22) - (3.24) in discrete time we analyzed the stability conditions of the linearized analog of these equations in continuous time and calculated the eigenvalues of the Lyapunov stability matrix.

Briefly, the calculation results can be summarized as follows:

- at conditions of the market price policy ($\sigma = 0$) gradual increase of parameter ε (in direction to an expansionary development policy) leads to significant differences in the growth rates of different processes, but the overall steady growth is maintained;

- at conditions of development policy focused on profitability (ε = 0), the gradual increase of parameter σ (*i.e.* transition to the pricing policy, which stimulates consumption) leads first to the drop of the growth rate, and then to the appearance of significant fluctuations of the processes intensities and prices, and at $\sigma \sim 0{,}5$ to the complete loss of stability and chaotization of the system;

- the simultaneous increase in σ and ε (the combination of the transition to the expansionist development policy and to the stimulating price policy) leads to a sharp slowdown in growth, but the loss of stability occurs much later (when σ and ε, become close to unity)[6].

As was shown above, in this model the development and pricing policy settings σ and ε significantly influence the pace and sustainability of economic dynamics. This line of research seems yo be far from exhausted, and especially

[6] These findings were confirmed by the eigenvalues analysis carried out on matrix of the right side of (20) - (21) derivatives with respect to y_i and p_j (Lyapunov exponents) witin the continuous-time model.

important in connection with the developing investigation of the reasons for the recent global economic crisis.

The above model can be used for simulating the non-agent environment within ADM approach, as shown below in Section 4.1. A closed system of $m+n$ dynamical equations describing the development of m individual processes, the intensity of which is managed on microeconomic level by different economic agents, and the equations of the dynamics of the "market" prices for n individual goods which are sold/bought by these agents, so that the general balance equations (1.3) and (1.4) are satisfied.

2.4. Model of economic system with an educational process

The extended technological model of economy (ETM) [20,24,25] described above in Section 2.1 takes into account the labor force factor and natural process of their reproduction. However, the qualification of labor force, as well as process of its improvement, has not been taken into account.

Below we consider a simple version of ETM with two categories of labor forces: 1) uneducated, having no special education, and educated [26]. Within this model these two types of labor force are represented by two goods that are marked accordingly LF_0 and LF_1.

Labor forces of different categories provide labor services, used by the production processes. Services are represented within this model as special goods. Unlike usual material goods there is no time lag for services in the material and financial balances (2.1) – (2.4). That means both provision and consumption of services are related to the same time period.

The basic assumptions of the model are:

- two categories of labor force – educated (LF_1) and uneducated (LF_0), provide two different types of labor services: LS_1 and LS_0;

- a person belongs to educated labor force, if it has any level of higher education;

- demographic indicators (fertility and mortality) and the normative ratio of students and teachers are constant;

- nobody can learn and work simultenously, or work at several positions;

- learning is described by single aggregate educational process;

- exists single aggregate production process and single aggregated material good.

Labor services measured in number of employees. Labor services of different categories are labeled LS_0 and LS_1 accordingly for LF_0 and LF_1.

For each category of labor forces: for LF_0 and LF_1, process of its reproduction exists. The costs of these processes are included into final consumption, and outputs are labor services and labor force.

Uneducated labor force LF_0 can get educated and pass to the LF_1 category. To provide this, an educational process is introduced in the model offering a new good called "educational services" (ES). This process utilizes both material goods, and LS_1 (teachers) and LF_0, as a raw material. The outputs of this process are educational services ES, measured in number of students. Besides ES the educational process produces LF_1, as the share some of those who attended (training lasts several periods) and uses LF_0, (not qualified yet). The teachers (workers in education) can only be LF_1.

Summarising, we provide the list of six goods and services of the model:

0) Uneducated labor force (LF_0);
1) Educated labor force (LF_1);
2) Material goods;
3) Services of uneducated labor force (LS_0);
4) Services of educated labor force (LS_1);

5) Educational services (ES).

The list of all four processes considered in the model is as follows:

0) Process of LF_0 reproduction;

1) Process of LF_1 reproduction;

2) Educational process;

3) Production process.

For all goods and services the vector of prices is written as:

$$\mathbf{p}(t) = \{p_0(t), p_1(t), \ldots, p_5(t)\}.$$

And as the labor force and are not sold, so their prices are zero $p_0(t)=p_1(t)=0$. And, as the goods are aggregated material all good material goods, $p_2(t)$ is a price index.

The vector of the processes intensity is:

$$\mathbf{y}(t)^T = \{y_0(t), y_1(t), y_2(t), y_3(t)\} \tag{2.19}$$

Here: $y_0(t)$ is the total number of non-qualified labor force LF_0, $y_1(t)$ is the total number of educated labor force LF_1, $y_2(t)$ is the total number of teachers, $y_3(t)$ is the total number of employed in the production process.

The input and output matrices of this model are written as:

$$\mathbf{A} = \begin{pmatrix} \alpha_0(t) & 0 & k & 0 \\ 0 & 1 & 0 & 0 \\ a_{20} & a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 & q \\ 0 & 0 & 1 & 1-q \\ 1-\alpha_0(t) & 0 & 0 & 0 \end{pmatrix}, \tag{2.20}$$

$$\mathbf{B} = \begin{pmatrix} \alpha_0(t)\cdot(1-\delta_0)+\beta_0 & \beta_1 & k\cdot(1-w)\cdot(1-\delta_0) & 0 \\ 0 & 1-\delta_1 & w\cdot k\cdot(1-\delta_0) & 0 \\ 0 & 0 & 0 & b_{23} \\ \alpha_0(t) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & k & 0 \end{pmatrix}. \qquad (2.21)$$

Here $\alpha_0(t)$ is the portion of economically active uneducated labor force during t; β_0 is the number of new uneducated labor force (thous who was born in the period (t-14) minus deceased), and β_1 is the same for educated; δ_0 mortality rate of uneducated and δ_1 is the same for educated labor force; k is factor indicating normative number of students per teacher, b_{23} is factor that indicates the output per worker, *i.e.* the average productivity;

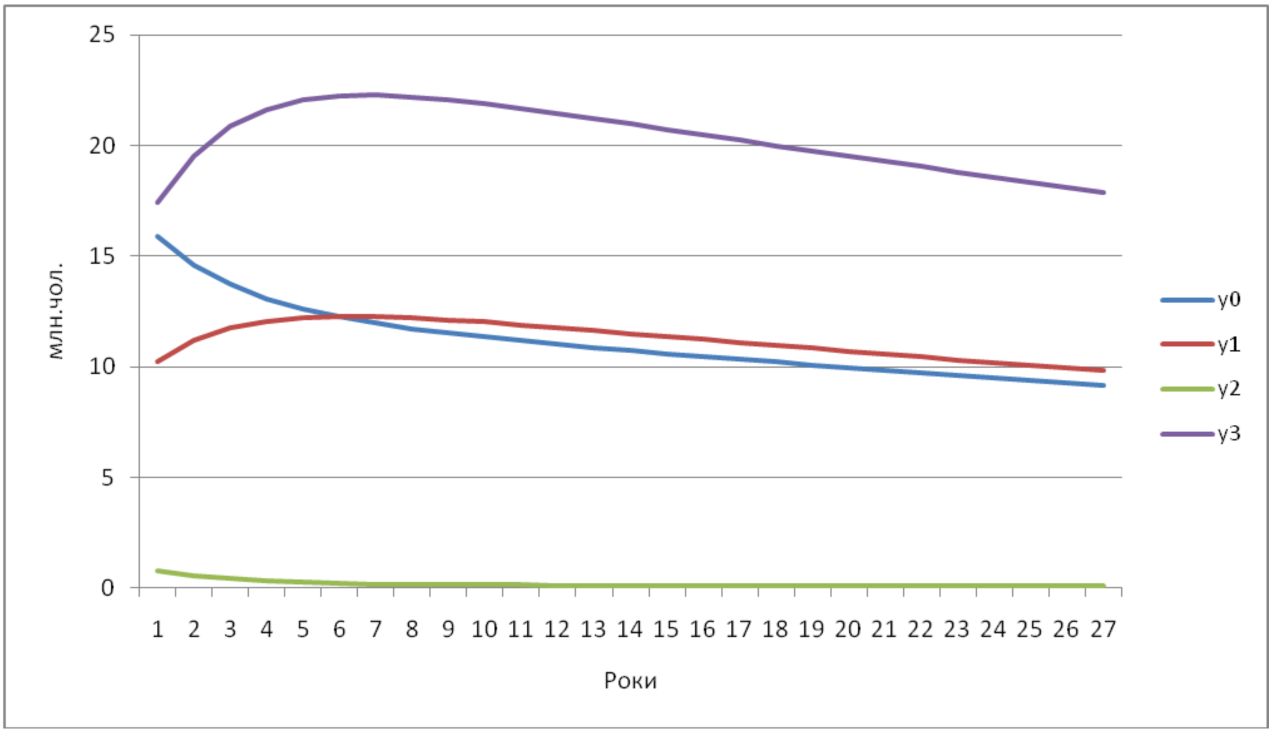


Fig.2.2. Dynamics of processes intensities for the equilibrium initial conditions.

The elements of input matrix a_{20} and a_{21} definie non-production consumption per person: a_{20} describes consumption per uneducated and a_{21} describes consumption per educated person. Element a_{22} defines consumption of material resources in educational process per teacher and a_{23} is the same in production process per employee;

Normative factor q indicates the share of uneducated LF_0 in production process. Finally, w is the share of graduated educated to the number of students. In general case, the matrix elements in (2.29) - (2.21) may be variable.

In [26] this model was applied to modeling of Ukrainian economy. Year 2008 was chosen as a baseline (t=1). It was accepted the full employment at t=1, and the labor force is fully employed in all processes. Part of LF_0 provides LS_0, the rest are students. Educated labor force LF_1 is distributed between the education and manufacturing processes.

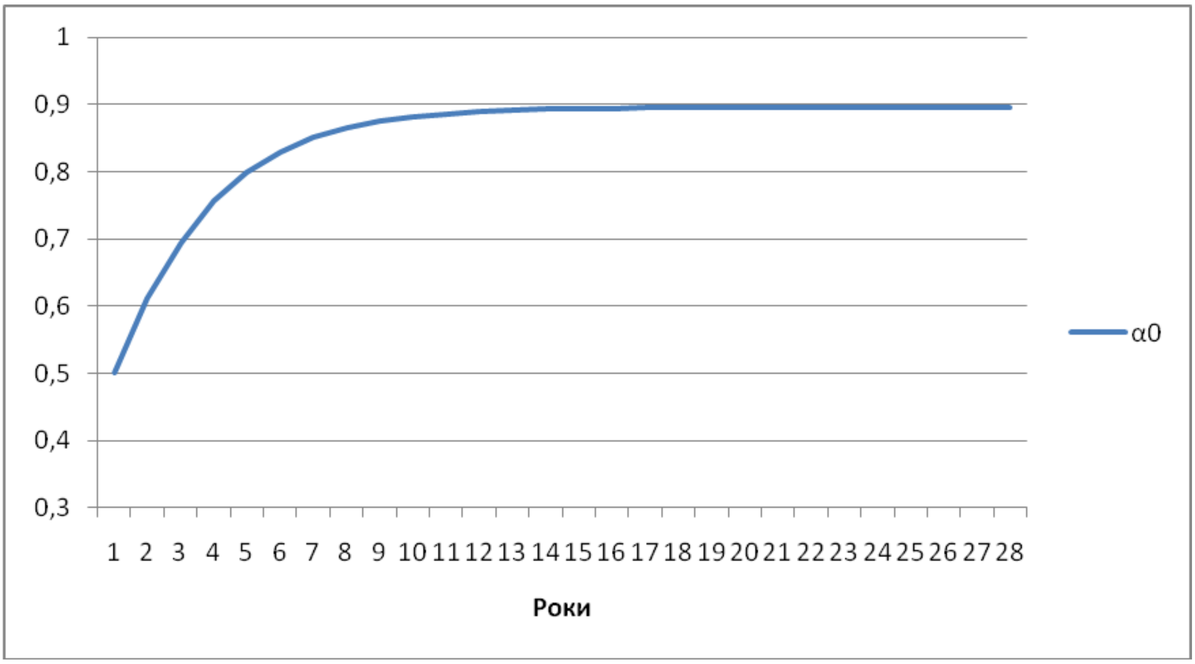


Fig.2.3 Dynamics α_0(t) for balance in the initial period

Given the above assumptions the technological matrices obtained in [26], are as follows:

$$\mathbf{A}=\begin{pmatrix} 0,766 & 0 & 10 & 0 \\ 0 & 1 & 0 & 0 \\ 35836,46 & 58021,14 & 49401,96 & 86160,16 \\ 0 & 0 & 0 & 0,457 \\ 0 & 0 & 1 & 0,543 \\ 0,234 & 0 & 0 & 0 \end{pmatrix}, \qquad (2.22)$$

$$\mathbf{B}=\begin{pmatrix} 0,760 & 0,0156 & 8,1579 & 0 \\ 0 & 0,9729 & 1,5331 & 0 \\ 0 & 0 & 0 & 137002,02 \\ 0,766 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \end{pmatrix}. \qquad (2.23)$$

According to (2.22)-(2.22) the economic dynamics is described by the following set of equations:

$$\begin{cases} y_0(t)=(\alpha_0(t-1)\cdot(1-\delta_0)+\beta_0)\cdot y_0(t-1)+\beta_1\cdot y_1(t-1)+k\cdot(1-w)\cdot(1-\delta_0)y_2(t-1); \\ y_1(t)=(1-\delta_1)\cdot y_1(t-1)+w\cdot k\cdot(1-\delta_0)\cdot y_2(t-1); \\ \alpha_0(t)\cdot y_0(t)=q\cdot y_3(t); \\ y_1(t)=y_2(t)+(1-q)\cdot y_3(t); \\ k\cdot y_2(t)=(1-\alpha_0(t))\cdot y_0(t). \end{cases} \qquad (2.24)$$

Unfortunately, the vector of the processes intensity, calculated for the base period from available statistical data [35] does not satisfy the basic set of equations (2.24) and therefore has to be corrected. The following equilibrium initial intensities (in millions of people) were accepted [26]:

$$y^*(0)=(15,865;\ 10,240;\ 0,791;\ 17,401). \qquad (2.25)$$

These conditions give the following initial portion of economically active uneducated labor force: $\alpha_0(0)=0,501$. Accordingly were changed the variable elements in matrices (2.21)-(2.22).

As it was noted, some elements of technological matrices of the model in general are not constant. Dynamics of such unsteady element $\alpha_0(t)$ - the share of economically active LF_0 is shown in Fig.2.3. Dynamics of the processes intensities determined from (2.24) is illustrated in Fig.2.4.

As one can see in Figs.2.11 and 2.12, the modeled economy eventually approaches a stationary balanced (turnpike) regime. Intensity of all processes began to decrease each period with the same rate of 0.988, with the rate of reduction of the total number of labor force.

The share of of economically active uneducated labor force is approaching a constant value $\alpha_0(t)$=0,896, which is higher than obtained from the actual data. It was checked whether the balance restriction (should not be consumed more goods than it was produced) if fulfilled [26].

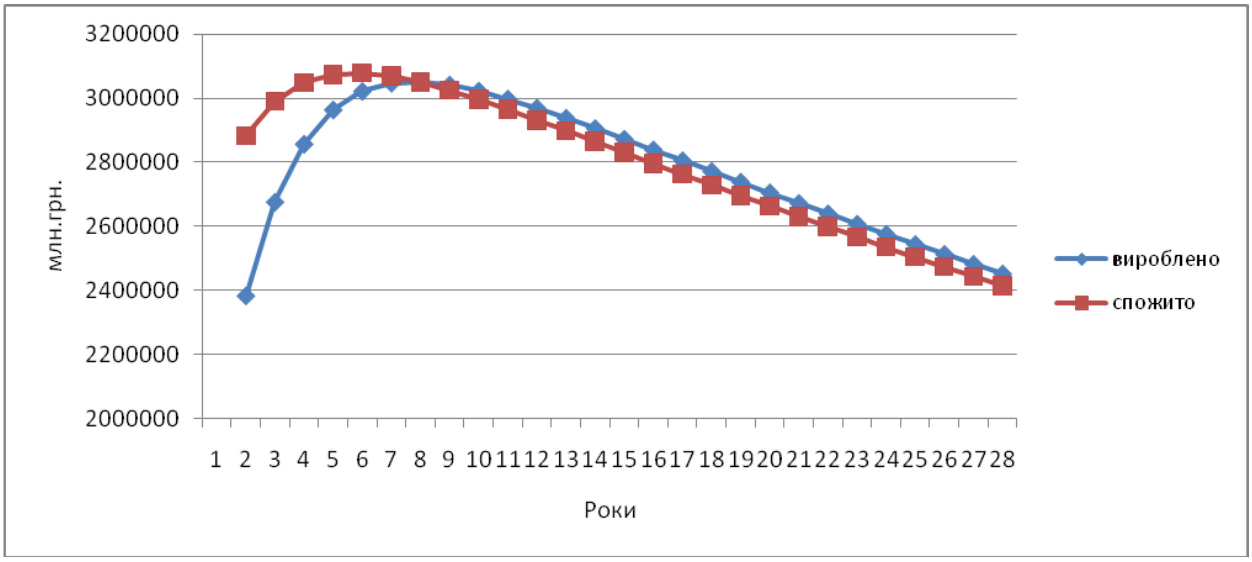


Fig.2.4. Production and consumption of goods for the equilibrium initial vector of intensities (2.25).

Since intensities were calculated from equations that provide a balance for the labor force, labor services and educational services, balance conditions for the relevant goods are met. A dynamics of production and consumption of the aggregate material good is presented in Fig.2.4.

From Fig.2.4 it is noticeable that during the first few periods the balance equation for material goods is not fulfilled. That is, consumed was more than it was produced. For the closed system it is unacceptable. But then the system approaches to the balanced development, and the material balance is regained.

In order to get on to the turnpike, system needs additional material resources from the outside, or must reduce its consumption to get enough its own resources. Eventually the system starts to develop in a balanced manner for all goods. And because intensities decrease each period, grows the portion of goods which are not consumed and can be exported if the system were open.

This model was investigated in [26] for stability (see Section 3.4 below). Instead of the equilibrium initial vector was used the actual vector of intensities. Similar results were obtained: the model system eventually gets on to the turnpike, where $\alpha_0(t)$ becomes constant and equal to 0.896. That is, the steady state of the model economy can be regarded as a stable one.

The model proposed in [26,27] can be used in the management on the national level, particularly in optimizing the parameters of the educational sector, as well as in modeling of the labor market. The turnpike development when achieved, is harmonizing the various processes in the economic system.

However, the more efficient model may be developed on the same basis, if methods of multi-agent modeling are invoked. This will allow taking into account many others factors influencing, *i.a.*, the distribution of labor force between the processes, including subjective factors (*e.g.* mentality).

2.5. Two-sector non-linear extended technological model of the real economy

As was already mentioned above, the extended linear technological model (ETM) of the idealized closed economy can be represented by *m* processes, producing and consuming *n* goods. Each process can consume and produce several goods, and their production per unit of (discrete) time is proportional to the so-called «intensity» of process, and all goods produced in the current period, must be be consumed in the next period.

Capabilities of such process to consume and produce goods is determined by the constant elements of technological input a_{ij} and output b_{ij} matrices, defining respectively, norms of consumption and production of *i*-th good in j-th process at its unit intensity, as described in Section 1.1.

Original von Neumann's model, being one of the most transparent and theoretically sound mathematical models in economicsdo not take into account, however, many factors that are essential in the real economies. Later there was proposed a number of modifications [9] and extensions of this model [24-27].

One of the most significant shortcomings of this model is absence in it any restrictions on the possible values of the variables. First, we should note the absence of restrictions on the possible values of the intensity of any process. Such traditional and natural economic factors as capital and labor force, are absent in it at all. Intensities in reality are limited by the production capacities, *i.e.* by fixed assets of enterprises and by availability of labor force. Within LTM these factors are considered, not only as unrestricted but also as free. On turnpike this leads to prices falling with the same rates, with wich the intensities grow.

Taking into account the realities of economy requires considering the limits of processes intensities depending on time due to 1) the depreciation of fixed assets, and 2) the availability of investments. Including of these factors makes the technological model at once non-linear.

The real economy is characterized by many other features, which are ignored in LTM as well. For example, factor of investment climate governing so-called induced investments, also is not present in this model.

In addition, von Neumann’s model is characterized by the absence of non-production consumption what separates the modeled economy from the society and leads to ignoring another important factor - the availability of labor resources. Often just this factor rather than available technologies, is limiting the economic growth.

At the same time ways to address all of these factors are well known and developed within the frameworks of other approaches (both Keynesian and neoclassical). The role of the capital depreciation and non-productive consumption was studied *e.g.* by Solow [28]. Effect of induced investments in formation of economic cycle was investigated by Hansen [29], Samuelson and Hicks [30] and others.

Using ADM it becomes possible to implement the ideas of these authors and overcome at least some of the above shortcomings of classical von Neumann’s model [4]. Below we consider an extended model reflecting features of a real economy, mentioned above, which takes into account the limitations of both the labor and industrial resources.

The first attempt to build an extended nonlinear technological model (NTM) of the economy – an improved version of von Neumann’s model, which takes into account limitations of manpower and production capacity, depreciation of fixed assets, the need in investments for expanded reproduction, was made in [31].

This work starts from an extended version of the model of von Neumann proposed earlier [20], which already includes the non-productive consumption and additional process of reproduction of human resources, as well the labor force as a specific "good" produced by this process.

Bearing in mind mainly a qualitative investigation of the proposed model capabilities, we restrict ourselves below to its simplest "two-sector" version, including only mentioned above "non-production" process, "producing" labor force (and marked below by subscript $_0$) and "manufacturing" process (index $_1$), which produces a single "material product" aggregating all goods produced in a real economy.

Within such two-sector model the column vector of process intensities consists of only two elements: $x_0(t)$ and $x_1(t)$:

$$\mathbf{x}(t) = \begin{pmatrix} x_0(t) \\ x_1(t) \end{pmatrix} . \tag{3.1}$$

First element of vector (3.1) has the following specific meaning: $x_0(t)$ is a measure of the "non-production" process intensity, and actually is a well-known demographic factor, proportional to the number of population at time *t*. Demographic dynamics is often considered to economics, and within this model it is quite natural. If we denote the natural population growth rate through λ, the production process intensity $y_0(t)$ will change with (discrete) time according to the law:

$$y_0(t+1) = (1+\lambda)\, y_0(t). \tag{3.2}$$

In turn $y_1(t)$ is a measure of the production process intensity. According to the concept of the extended technological model [20], the intensity of the production process is iust measured by the number of its employees (labor force).

The elements of the vector-row of prices

$$\mathbf{p}(t) = \begin{pmatrix} p_0(t) & p_1(t) \end{pmatrix} \tag{3.3}$$

are:

- $p_0(t)$ is «price» of the labor force, the of the “good” provided by "non-production" process, i.e. wages;

- $p_1(t)$ is a "price" of the aggregated material product.

It should immediately be noted that both concepts as "price" of the aggregate good and "price" of the labor force witin the framework of two-sector model are very conditional. These "pricees" should be understood simply as indices of prices and wages, necessary to compare their dynamics.

Naturally, also the non-negativity of all intensities and prices has to be assumed:

$$y_0(t) \geq 0; \quad y_1(t) \geq 0; \quad p_0(t) \geq 0; \quad p_1(t) \geq 0. \quad . \tag{3.4}$$

Since the non-productive sphere does not produce any material goods, and the production process does not produce labor force, in the output matrix only the diagonal elements are nonzero:

$$\mathbf{B} = \begin{bmatrix} b_{00} & 0 \\ 0 & b_{11} \end{bmatrix} . \tag{3.5}$$

Here b_{00} is demographic technological factor, *i.e.* the proportionality coefficient between the intensity of non-producting process $x_0(t)$ and of the labor force "output". The product $b_{00} \cdot x_0(t)$ is interpreted as the maximum amount of labor force, which can be employed in the manufacturing sector in the next period.

Element b_{11}, in turn, determines the output of the aggregate good per worker, *i.e.*, labor productivity. Because productivity depends on the capital intensity $k(t)$, the labor productivity value should be considered as a variable $b_{11} = b_{11}(t)$. Thus, to calculate the dynamics of the system output one must also have the equation for calculating the capital intensity as function of time: $k = k(t)$.

Input matrix in equations (3.1) and (3.2) is written in the form:

$$\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} . \tag{3.6}$$

Its elements are:

- a_{00} is the share of working population, which is not employed in the production of goods. It includes not only the unemployed but also those who are engaged in education, trade, *etc*. Generally speaking, this share depends on many factors, and there is no reason to believe that it is constant. To the definition of $a_{00} = a_{00}(t)$ we will return below;
- a_{01} is coefficient characterizing the involvement of labor force in non-productive process at its unity intensity. It is natural to set $a_{01} = 1$;
- a_{10} is per capita consumption of aggregated good in the non-production sphere. This value is, generally speaking, a variable: $a_{10} = a_{10}(t)$. It depends on the so-called "average propensity to consume" and will be also discussed below;
- a_{11} is specific industrial consumption of the aggregated good (per person employed in its production). This is a purely technological parameter of the model. If technical progress is not taken into account, the a_{11} value can be regarded as a constant.

We assume that the dynamics of prices and intensities is given by standard ETM equations (2.1) - (2.4) for a balanced economy. The values of the intensity of the production process $y_1(t)$ and labor productivity $b_{11}(t)$ define the output Y = $Y(t)$ of the aggregated good in physical terms (in units of the aggregated goods). The output Y(t), in turn, is

$$Y(t) = b_{11}(t) \cdot x_1(t) . \tag{3.7}$$

It is also necessary to consider the possibility of external investments, that can be specified by the vector $\mathbf{I}(t) = [I_0(t)\ I_1(t)]$. The latter, in turn, will be considered as induced, *i.e.* proportional to the increase in output of the current period in respect to the previous period. So, we can write [30] for investment in the production sector:

$$I_1(t) = \rho\left(Y(t) - Y(t-1)\right). \tag{3.8}$$

Here ρ is a certain numerical characteristic of the investment attractiveness. Each investment is considered in the model as an external and, accordingly, should be linked to the payment of interest at a rate of ***r***.

In this chapter we restrict ourselves to the description of a balanced economy, *i.e.* we assume the material balance equation:

$$\mathbf{A}\cdot\mathbf{y}(\mathrm{t}+1) = \mathbf{B}\cdot\mathbf{y}(\mathrm{t}) \quad . \tag{3.9}$$

Financial balance, which determines the ratio of costs and revenues for each of the processes of two consecutive periods of time, must include the interest payments on investments of the previous period. Per unit of labor force it gives:

$$\mathbf{p}(\mathrm{t}+1)\cdot\mathbf{B} = \mathbf{p}(\mathrm{t})\cdot\mathbf{A} + r\cdot\mathbf{i}\left(t-1\right) . \tag{3.10}$$

Here $\mathbf{i}(t) = \mathbf{I}(t)/y_1(t)$ is row vector of the specific investment (per person employed in the production process). For simplicity, we will restrict considering such investments only in production sector: $\mathbf{i}(t) = \left(0, I_1(t)/y_1(t)\right)$.

Multiplying vectors and matrices in the equations of material and financial balances (3.9) and (3.10), we arrive at the following system of four equations:

$$\begin{cases} a_{00}(t+1)\cdot y_0(t+1) + y_1(t+1) = b_{00}\cdot y_0(t) \\ a_{10}(t+1)\, y_0(t+1) + a_{11}\cdot y_1(t+1) = b_{11}(t)\cdot y_1(t) \\ b_{00}\cdot p_0\left(t+1\right) = p_0\left(t\right)\cdot a_{00}\left(t\right) + p_1\left(t\right)\cdot a_{10}\left(t\right) \\ p_1\left(t+1\right)\cdot b_{11}\left(t-1\right) = a_{11}\cdot p_1(t) + p_0(t) + r\cdot i\left(t-1\right) \end{cases} \tag{3.11}$$

Each equation of the system (3.11) has a clear meaning. The first equation defines the labor force balance, "produced" in non-productive process in the current period and then been "consumed" or remain unemployed in the next period.

The second equation is the condition of the aggregated material goods balance. It shows how the output of the current period is divided between non-productive and production consumption of the next period.

The third equation is a restriction which ensure solvent demand of the population (workers' wages must be sufficient to ensure the non-productive consumption of the whole population). This equation determines the price of the consumption of aggregated good, which must be specified for the next period to reimburse the industrial consumption costs, labor costs and interest payments on the investments of the previous period.

In order to make the system of NTM equations (3.11) closed, we must add an equation describing population dynamics (3.4) and define the time dependence of three variables: capital intensity $k = k(t)$ and two elements of matrix **A:** $a_{00}(t)$ and $a_{10}(t)$. Only after that NTM can be applied to the calculation of the dynamics of the intensities, output and prices.

To implement it, we need an equation for capital dynamics. Following Solow [28], the time dependence of the capital value *K*(*t*) can be represented as follows:

$$K(t+1) = (1-c)Y(t) + (1-\mu)K(t). \qquad (3.12)$$

Here *Y*(*t*) is the output of the current period (in value terms); parameter c is so-called propensity to consume $(0 < c < 1)$, and μ is the capital depreciation rate $(0 < \mu < 1)$.

After adding the above-mentioned external induced investments $\mathbf{i}(t-1)$ (3.8) the capital dynamics equation takes the form:

$$K(t+1) = (1-c)Y(t) + (1-\mu)K(t) + \rho\big(Y(t) - Y(t-1)\big). \qquad (3.13)$$

Because outputs of the current and previous periods depend on the value of capital $K(t)$ (3.7) and labor force $y_1(t)$, it is necessary to have an explicit expression for the production function

$$Y(t) = Y\left(K(t), x_1(t)\right). \tag{3.14}$$

This makes the system of equations (3.4), (3.11) and (3.12) closed.

With a view to arrive at a model whose parameters have definite economic sense, we will further to use Leontief production function [32], representing the output as a function of two factors - capital K and labor L as follows:

$$Y = \min\left(\frac{K}{K_{\max}}, \frac{L}{L_{\max}} \right), \tag{3.15}$$

where $K_{\max}$ and $L_{\max}$ are parameters of Leontief production function. Dividing both sides of (3.15) by $L = x_1(t)$, and taking into account definition (3.7), we obtain the following expression for labor productivity as a function of time $b_{11} = Y(t)/L(t)$:

$$b_{11}(t) = b_{\max} \cdot \min\left(\frac{k(t)}{k_{\max}}, 1 \right). \tag{3.16}$$

Here $b_{\max} = 1/L_{\max}$ is a constant which has the meaning of maximum labor productivity, corresponding to the given capital intensity $k(t) = K(t)/L(t)$, not exceeding $k_{\max} = K_{\max}/L_{\max}$ - the maximum capital intensity at the current level of capitalization.

It is essential to note here that using relations based on different approaches within one calculation scheme, can complicate it due to possibility of the presence in it parameters that cannot be regarded as independent. Here it refers primarily to the simultaneous use in the same scheme of the matrix element $a_{10}(t)$ and other parameter, the average propensity to consume $\boldsymbol{c}$. The function $a_{10}(t)$ determines the amount of non-productive consumption of the aggregated material good and, therefore, should be closely related to the average propensity to consume - namely, to the ratio of non-productive consumption of the current period to the previous output must be equal, by definition, to the average propensity to consume.

Furthermore, since demographic dynamics is already given by the equation (3.4), the value $a_{00}(t)$, reflecting the number of potential labor force, not employed in the production sector can be calculated from the first equation (3.11), and the ratio of industrial consumption of the current period to the output of the previous should be equal to the average propensity to save *s* = (1 - *c*). Given the above conditions, we can now write the following equation of the labor force dynamics:

$$y_1(t+1) = (1-c)\frac{b_{11}(t)}{a_{11}}\cdot y_1(t)\,; \tag{3.17}$$

express the value of $a_{10}(t)$ from the second equation of (3.11) system:

$$a_{10}(t+1) = c\cdot b_{00}\cdot\frac{b_{11}(t)}{(1+\lambda)^2}e(t); \tag{3.18}$$

and derive the equation for $a_{00}(t)$ from the first equation of (3.11) system:

$$a_{00}(t) = \frac{b_{00}}{1+\lambda}\left(1-e(t)\right)\,. \tag{3.19}$$

Here $e(t)=\dfrac{x_1(t)}{b_{00}\cdot x_0(t-1)}$ defines the level of employment ($e(t)\le 1$).

It should be noted that the above set of equations (3.17) - (3.19) is valid only in the case of unsaturated labor market, *i.e.* when the need of production sector in the labor force does not exceed the number of working population: $e(t)<1$. If this condition is violated, then the labor market becomes saturated, *i.e.* employment in the production sector turns to full: $e(t)=1$. The intensity of the production process in this case is limited not by its capitalization but by the available labor force:

$$x_1(t) = b_{00} \cdot x_0(t-1) . \tag{3.20}$$

Under this condition $a_{00}(t)$ becomes zero, and the function $a_{10}(t)$ takes the form:

$$a_{10}(t+1) = b_{00} \cdot \left(\frac{b_{11}(t)}{(1+\lambda)^2} - \frac{a_{11}}{1+\lambda} \right). \tag{3.21}$$

In the case of unsaturated labor market ($e(t)<1$), the system of equations of the model (3.4), (3.11), (3.13), (3.18), (3.19) and (3.16) can be solved and intensities and prices of the next period can be explicitly expressed in terms of the corresponding values of the previous period.

Price of the labor force (index of wages) of the next period $p_0(t+1)$, if we know current values of the intensity of the production process (labor force $x_1(t)$), employment $e(t)$, and prices on the aggregated material goods $p_1(t)$, can be found from equations (3.11):

$$p_0(t+1) = \frac{1}{1+\lambda}\Big[\big(1 - e(t)\big)\, p_0(t) + c \cdot b_{11}(t) \cdot p_1(t)\Big]. \tag{3.22}$$

Note that due to need to ensure solvent demand, *i.e.* conditions defined by the third equation of the (3.11) system, wages are proportional to both the propensity to consume, and the current price of the aggregated good.

The price of the aggregated good in turn is calculated from the latter equation of (3.11) system with account of (3.19):

$$p_1(t+1) = \frac{a_{11}}{b_{11}(t+1)} \cdot p_1(t) + \frac{p_0(t)}{b_{11}(t+1)} + r \cdot \rho \cdot \frac{Y(t-1) - Y(0)}{x_1(t)\, b_{11}(t+1)} . \tag{3.23}$$

As we can see here, along with the classical von Neumann contribution related to the cost of the production consumption (first term), there are two contributions determined by the necessity of reimbursement of wages (second term) and the payment of interest on investments (last term).

System of equations (3.4), (3.17), (3.22) and (3.23) for intensities and prices can be closed by equation for the capital intensity, following from (3.13) and (3.14):

$$k(t+1) = a_{11}\left[1+\frac{(1-\mu)\cdot k(t)}{(1-c)\cdot b_{11}(t)}+\frac{\rho}{1-c}\left(1-\frac{x_1(t-1)\cdot b_{11}(t-1)}{x_1(t)\cdot b_{11}(t)}\right)\right]. \tag{3.24}$$

Thus, all the model variables can be evaluated sequentially as functions of discrete time, if their initial values are known.

For practical application only a few constants having the following clear economic sense must be defined:

- a_{11} - productive consumption per unit of labor force;
- μ - depreciation rate;
- c - the average propensity to consume;
- ρ - investment attractiveness;
- r - rate of interest;
- b_{max} and k_{max} - parameters of the labor productivity function (3.16).

All these model parameters take positive values only. Below we present and discuss a few examples of the proposed model application. We consider three characteristic modes of economic dynamics, and see how the poposed model reproduces them. The numerical values of the parameters corresponding to these three modes are listed in Table 3.1.

In all cases the initial values are assumed equal to be the same, presented in the last column of the Table 3.1 ($x_0(0)=x_1(0)=p_0(0)=p_1(0)=k(0)=A_0$).

Table 3.1.

Parameters corresponding to the different dynamic regimes of NTM

Fig. No.	c	R	μ	λ	a_{11}	b_{max}	k_{max}	ρ	b_{00}	Initial value (A_0)
3.1	0.700	0.01	0.01	0.02	0.2	0.7	0.7	1.0	0.7	0.25
3.2	0.885	0.1	0.1	0	0.6	6.0	6.0	1.0	0.7	0.25
3.3	0.885	0.1	0.1	0	0.6	5.92	5.7	1.0	0.7	0.25

The first question that arises in the study of economic dynamics within any model of real economy is the ability to describe its sustainable economic growth under conditions of stable full employment and minimum capitalization sufficient for given rate of production growth. If we take this rate of growth equal to the growth rate of labor force, such a "natural turnpike" mode corresponds to the following two conditions.

1) The growth rate of the economy is equal to the growth rate λ of labor force**:**

$$x_1(t+1) = (1+\lambda)\, x_1(t)\ . \tag{3.25}$$

2) Capital intensity and labor productivity are constant and equal:

$$k(t) = k_{\min}; \quad b_{11}(k(t)) = b_{11}(k_{\min}) = b_{\max}\ . \tag{3.26}$$

Accordingly, the output of material goods is written in the following form $Y(t) = b_{\max} \cdot x_1(t)$, and the requirement of stable employment leads to the fact that the average propensity to consume must comply with the following condition:

$$c = 1 - (1+\lambda)\frac{a_{11}}{b_{\max}}\ . \tag{3.27}$$

At the same time, the behavior of prices in such a "turnpike" mode (3.25) - (3.26) is significantly different from the predictions of the von Neumann’s model.

Thus, if we ignore the initial value of output $Y(0)$, the equation (3.23) within this mode becomes:

$$p_1(t+1) = \frac{a_{11}}{b_{\max}} p_1(t) + \left(\frac{1}{(1+\lambda)^2} - \frac{a_{11}}{(1+\lambda) b_{\max}} \right) p_1(t-1) + \frac{r \cdot \rho}{1+\lambda} . \qquad (3.28)$$

As one can see, the price of the aggregated good (3.28) and, accordingly, the wages (3.22) in this case are described by the equation of the second order dynamic element and may either fall or rise and even oscillate depending on the initial conditions and technological parameters.

In particular, they can be stabilized with a non-zero rate of interest and investment attractiveness. Then the values of stable levels of wages $\bar{p}_0$ and prices $\bar{p}_1$ can be found from equations (3.22) and (3.28):

$$\bar{p}_0 = \frac{r \cdot \rho}{\lambda(1+\lambda)} + \frac{r \cdot \rho}{\lambda\left(a_{11}(1+\lambda) - b_{\max}(2+\lambda)\right)}; \quad \bar{p}_1 = \frac{r \cdot \rho}{\lambda} \left(\frac{2+\lambda}{1+\lambda} - \frac{a_{11}}{b_{\max}} \right)^{-1} . \qquad (3.29)$$

The way of reaching such a regime, as in the turnpike theories, to a large extent depends on the initial conditions and model parameters. With the average propensity to consume and investment attractiveness (or the rate of interest) tending to zero, prices actually fall at the same rate with which the economy grows, as in von Neumann’s model.

However, three examples provided below illustrate existence of substantially different modes of economic dynamics described by the NTM model [31,34].

Fig. 3.1 represents the first such example: the approaching of the "turnpike mode". It corresponds to the first set of model parameters from the Table 3.1. First, there is a sharp drop in employment and significant fluctuations in prices and wages, which gradually turn into increase in employment.

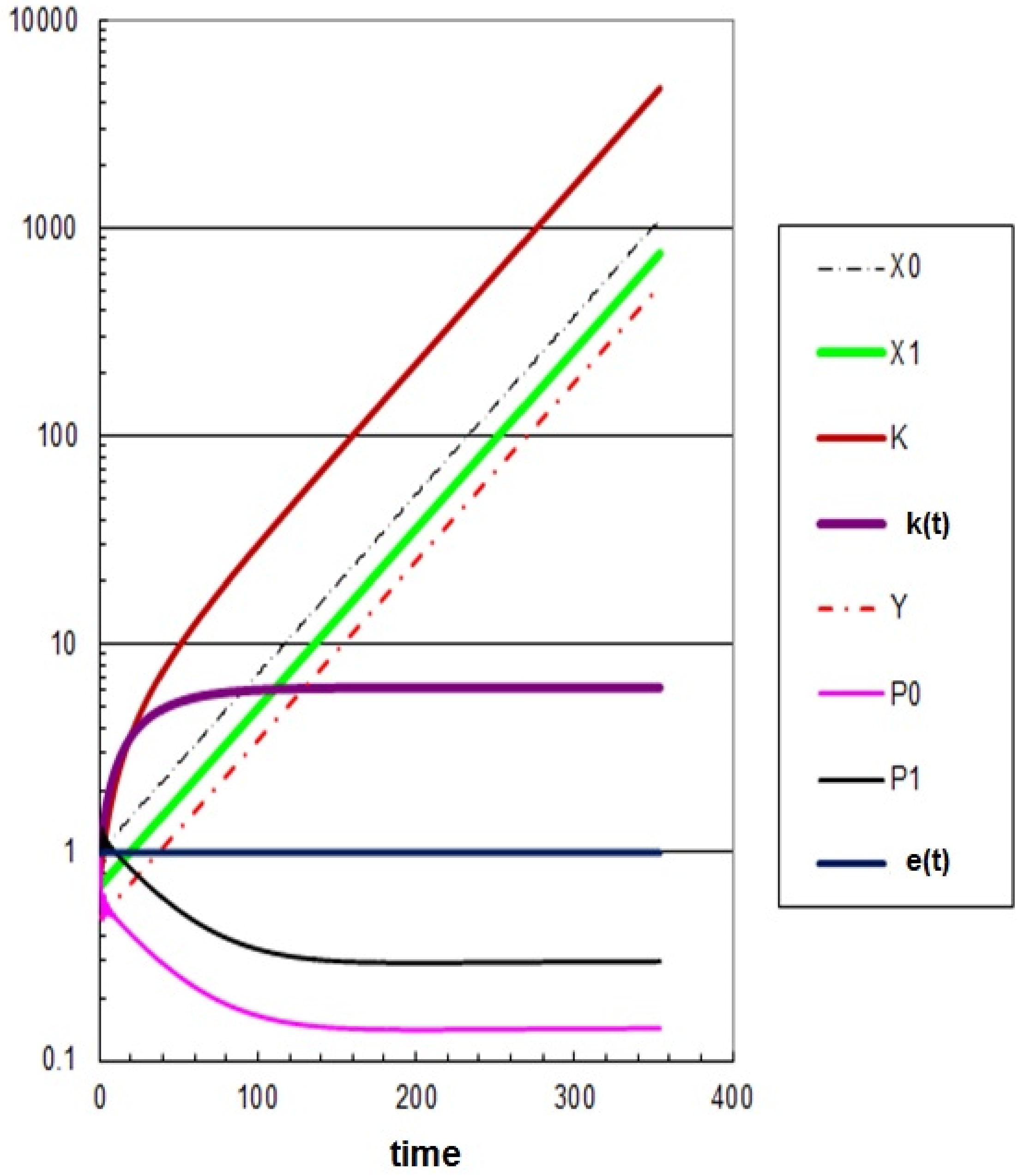


Fig. 3.1. Approach to the "turnpike mode" (the first set of parameters in the Table 3.1.)

After t = 100 the full employment is achieved and in the future there is only the "turnpike" growth at a rate determined by the rate of population growth, with the prices, wages and capital-labor ratio gradually stabilizing.

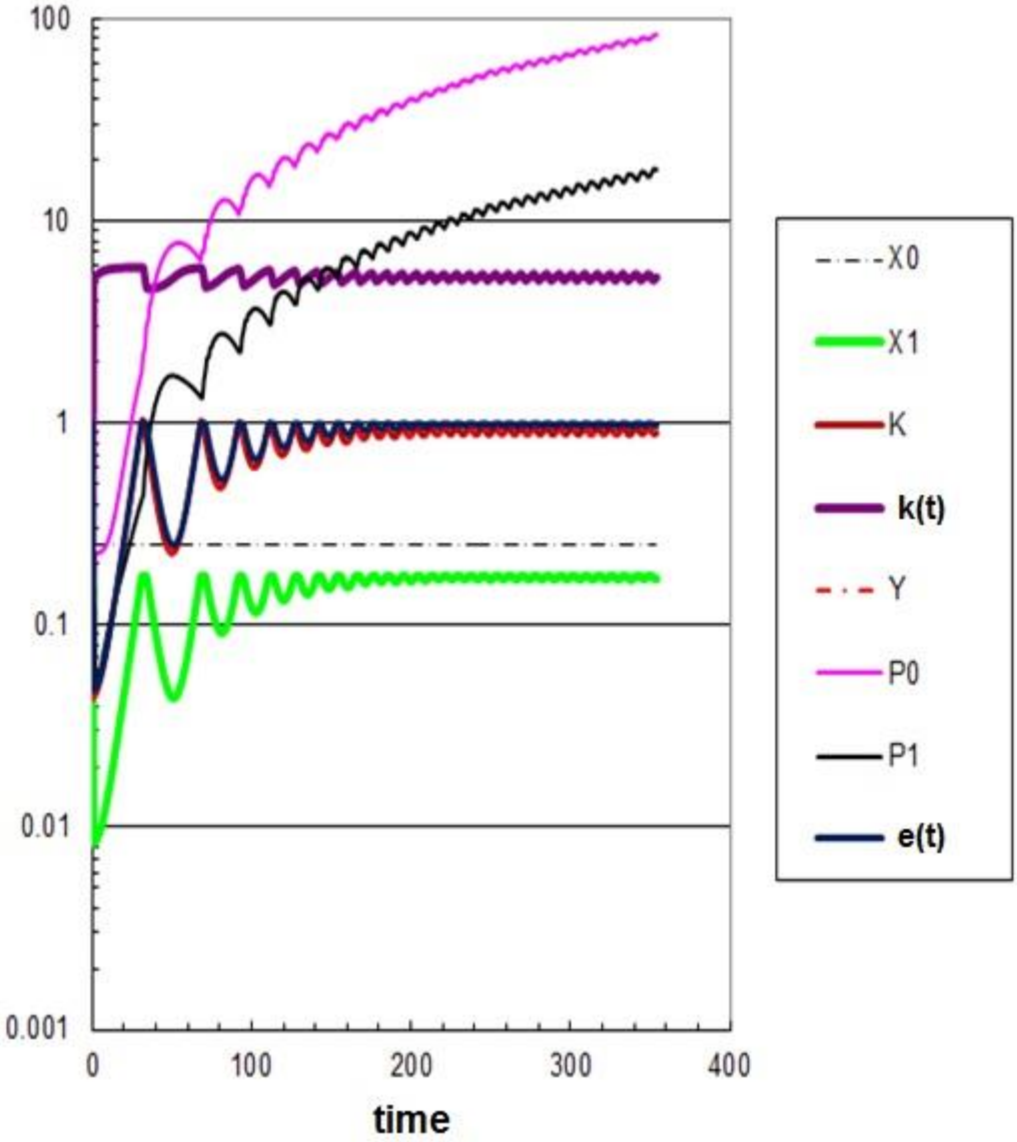


Fig. 3.2. Stagflation with the damped cycle at full employment (second set of parameters in Table. 3.1)

The second set of parameters in Table 3.1 corresponds to the zero rate of population growth, full employment but also to the high interest rate. However, this regime, illustrated in Fig. 3.2, is also non-monotonic: output, capital, prices and wages fluctuate greatly at first. Then the stagnation, accompanied by a significant rise in prices ("stagflation"), is achieved.

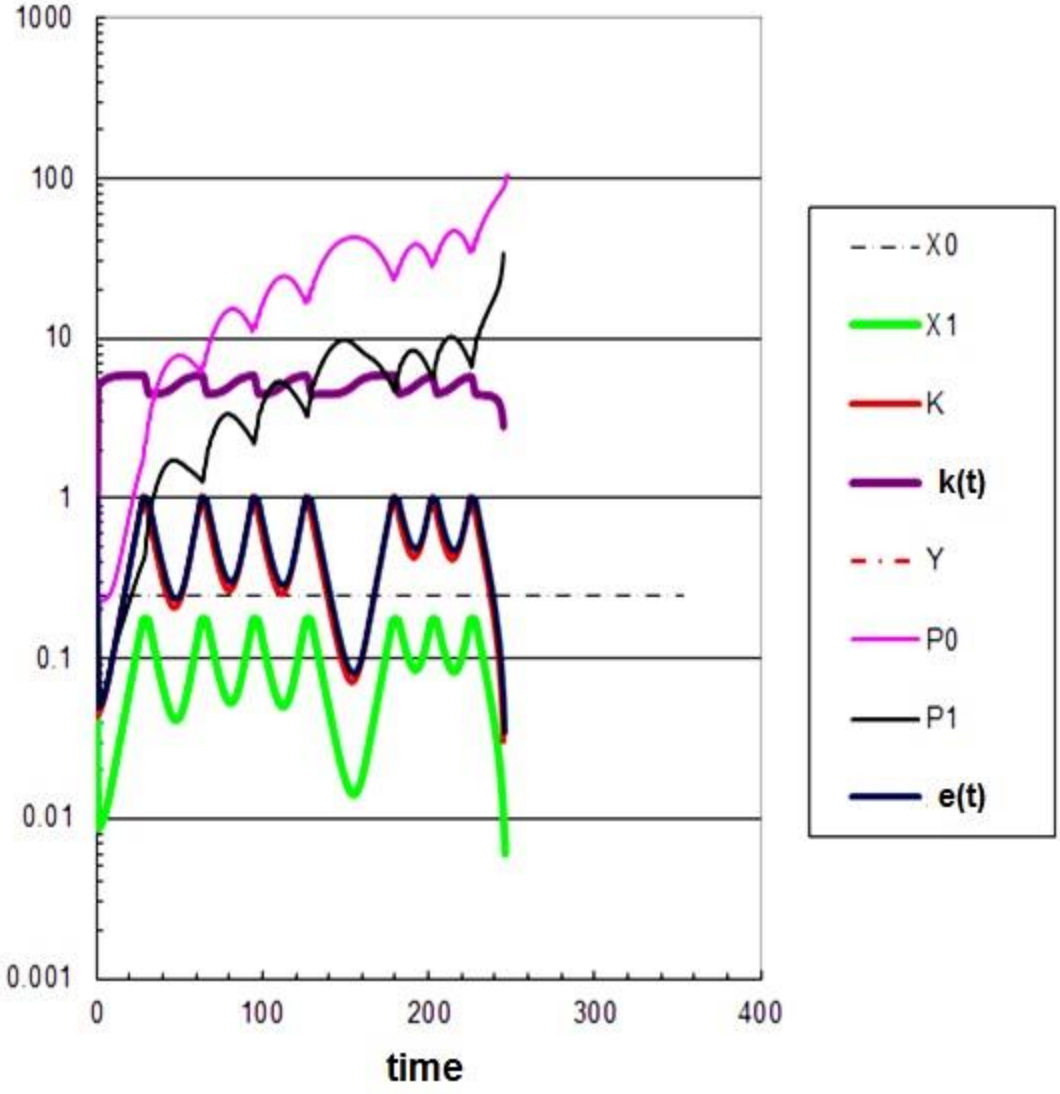


Fig. 3.3. The emergence of non-linear economic cycle and the crisis (the third set of parameters in Table 3.1.)

The third set of parameters in Table 3.1 illustrates the possibility of nonlinear economic cycle and crisis at a stable population. Here, the initial values prices, wages and the intensities are reducing, and the propensity to consume and the rate of interest are high. Despite it due to falling of the initially high value of output foreign investments are absent. Full employment at low prices and wages is periodically achieved. The amplitude of the nonlinear oscillations grows and at t = 300 the crisis occurs.

As seen in the Fig. 3.3, with prices soaring, the capitalization and employment are quickly falling and the economy fully stops after 250 periods.

It should be noted that such a crisis arises only at certain combinations of the model parameters. In the case shown in Fig. 3.3, a very small (less than 1%) decrease in $b_{\max}$ parameter, for example, leads to the disappearance of critical phenomena, and its small increase leads to the much earlier development of crisis.

The detailed study of the conditions of occurrence of such "critical point" and sustainability of the NTM is of great interest in itself however, it goes far beyond the scope of this work.

It should be emphasized the prospects of combination of the NTM approach with equations of microeconomic dynamics of individual processes that are compatible with the balance equations (1.3) - (1.4) von Neumann, described in Section 2.3. This approach allows simulating the effect of development policy and prices policy factors on the real macroeconomic dynamics. In today's global and significantly monopolized economy both these tools are used to achieve the long-term goals and not always the "objectives" of the process owners (*e.g.*, corporations) are not simply profit-oriented in the short term. Therefore, the modification the NTM, which takes into account the possibility of deviation from market principles, is fully justified.

Briefly summarizing the presented above, it should be emphasized that the above results confirm the generally accepted conclusion that the approach to the modeling of the economy proposed by von Neumann, is a good theoretical basis for construction of the applied dynamic models.

The method used to account for the characteristics of of labor resources within the NTM is, of course, simplistic and does not take account of certain features of reality, for example, it does not reflect the structural changes in total employment, differential pricing of labor resources, depending on qualifications etc. However, in the context of the extension of technological models it could easily be developed and used in research related to the dynamics of the labor force. The proposed NTM is alike the ETM (see Section 2.4) and can be extended by inclusion of "educational process" as a part of the non-productive sphere, to study effect of training and retraining of labor resources in the economy.

Approach applied in development of NTM is not limited to a closed economy. Expansion of NTM on an open economy, allows taking into account exports and imports, as well as to reflect another type of balance - the balance of foreign trade as it was demonstrated on LTM in Section 2.2.

3. Dynamic models of the real economy

3.1. Technological model of Ukrainian economy

Unfortunately, using of LTM containing all the production processes of the country in practice is not possible. No statistical source is available providing complete information about all processes that operate in a country.

As an alternative, the State Statistics Committee of Ukraine annually publishes the input-output table [35], based on Leontief model [32]. The latter is an aggregated model related to the von Neumann’s LTM [55] and can be used as a basic one in construction of approximate technological matrices within ETM.

The purpose of this Section is to discuss an approach to construction and investigation of such approximate version of the Ukrainian economy LTM [36]. We will formulate principles of transformation of the existing intersectoral balance (ISB) tables to the input-output matrices and of construction on this basis of the LTM technological matrices, as well as vectors of intensities and prices, and explore to study the possibilities for economic development of Ukraine to get on to the turnpike.

When passing from one model to another one must take into account their major differences, namely:

– in Leontief "input-output" model the ISB is built "year by year" without a time lag, and in LTM there is a lag of one period between output and consumption;
– input-output model and ISB is build for an open economy, *i.e.* takes into account external flows of goods and the basic LTM model is valid for the closed economy;
– consumption of population wihin ISB is considered separately and in the basic LTM consumption of workers is the part of processes input.

Taking into account these differences, the Ukraine's ISB table of 2008 was first rebuilt in aqs follows.

In the ISB table were selected 15 pure industries: agriculture, mining, construction and others. Separately was added a supporting column "Payment of financial intermediaries", which containes the only non-zero number in its 10th line "Financial activities". These costs were considered as public spending and added to the "Final consumption expenditure" on the same line. Thus, the 16-th column of the main part of ISB was removed from the table.

The values in the first quadrant ("Intermediate consumption") labeled x_{ij} - are amounts of goods of industry i, consumed in industry j in the current period and the gross output of industry j is designated as X_j.

In the second quadrant of the ISB table, are written final consumption, gross export and import of goods and services. For simplicity the gross accumulation was attributed to final consumption, and the sum of these columns was marked as FP_i (final good of *i*- th industry).

To separate the output of domestic industries, the imports (Im_i) were proportionally deducted from the elements of intermediate FP consumption in every line. New items are calculated as follows:

$$x2_{ij} = x_{ij} - Im_i \cdot \frac{x_{ij}}{\sum_k x_{ik} + FP_i}, \quad FP2_i = FP_i - Im_i \cdot \frac{FP_i}{\sum_k x_{ik} + FP_i}.$$

Here import is distributed simplistic, *i.e.* proportionately. More accurate distribution requires additional statistics.

External production system was introduced to the model as a separate 16-th process. In the 16-th line was written the total cost of imported goods of all categories of the industry and population ($\sum_i (x_{ij} - x2_{ij})$ and $\sum_i (FP_i - FP2_i)$), and "export" appears in the 16-th column as an external input of the process.

In the third quadrant of ISB table, which is called below 'intermediate consumption', are written taxes, wages, profits *etc*. The sum of these lines data gives the "Gross Domestic Product" (GDP_j). The latter was treated as household income spent on final consumption.

The difference emerged between the amount of FP and GDP, and consequently between the gross output and consumption (75.729 billion UAH) is the export-import balance. For Ukraine it is negative. It turns out that there was more consumed than was earned.

Suppose such a situation did not arise, and the difference is covered by incomes from abroad. This difference is included to the line with the GDP, as income from exports, *i.e.* the 16-th column gives GDP_{16} = 75.729 billion UAH.

In the underlying LTM model workers are considered as a part of the manufacturing process, so their income and expenses are added to the corresponding income and expense of the corresponding process. So, FP should be prorated between the elements 'of intermediate consumption'. The share of GDP will servr here as a weighting factor. So we have:

$$\hat{x}_{ij} = x2_{ij} + FP2_i \cdot \frac{GDP_j}{\sum GDP_j}, \tag{3.30}$$

where $\hat{x}_{ij}$ (i=1,…,16; j=1,…,16) specifies the elements of the transformed ISB.

ISB obtained in such manner has balance amount greater on the amount of imports than the original one. This is due to that fact the output of external system was taken into account along with output of the internal (domestic) processes.

If we look at ISB in terms of the basic LTM, it appears that in this table the column "Use of goods and services" reflects the output of the previous period (year) for each sector in current year prices, *i.e.*:

$$X_j(t) = \sum_{i=1}^{16} p_i(t) \cdot b_{ij} \cdot y_j(t-1),\ j=1,\dots,16,$$

where $p_i(t)$ is the price of good i, b_{ij} are elements of the output matrix, $y_j(t)$ is the intensity of j-th process.

However, this expression becomes simplier, because ISB considers here only pure industries, and if we know prices of the current year and intensities of the previous one, we can calculate elements of the diagonal output matrix **B**:

$$b_{jj} = \frac{X_j(t)}{p_j(t) \cdot y_j(t-1)}. \tag{3.31}$$

All non-diagonal elements of the output matrix **B** are zero.

On the other hand, the main quadrant data in the ISB table reflect the current year consumption (expenditure) of industries:

$$\hat{x}_{ij}(t) = p_i(t) \cdot a_{ij} \cdot y_j(t),$$

were a_{ij} are elements of the input matrix **A**:

$$a_{ij} = \frac{\hat{x}_{ij}(t)}{p_i(t) \cdot y_j(t)}. \tag{3.32}$$

Suppose there is no difference between workers' wages, and the labor market at base period is in equilibrium. Because the employed labor force (HR) within LTM determine the intensity of processes, one can calculate the number of labor force in each process as follows:

$$y_j(t) = N(t) \cdot \frac{z_j(t)}{\sum_k z_k(t)}. \tag{3.33}$$

Here N(t) is total manpower over the country, and z_j (t) are the expences of j-th process on labor force.

In the final assessments years 2007 and 2008 were taken as baseline. For simplicity reasons the total amount of labor force was replaced by the economically active population: N(2007) = 22,322 and N(2008) = 22,397 (in millions) [35].

Labour costs are available in ISB for these two years. For the 16-th process, this figure was estimated as a positive export-import balance. According to (3.33) the following vectors of intensity were obtained:

$\mathbf{y}(2007)^T$ = {1,386; 0,011; 1,040; 5,727; 0,825; 1,033; 2,743; 0,221; 2,145; 1,209; 1,846; 0,954; 0,955; 0,654; 0,402; 1,172};

$\mathbf{y}(2008)^T$ = {1,423; 0,008; 1,311; 5,183; 0,751; 0,791; 2,800; 0,242; 2,020; 1,461; 1,855; 0,951; 0,952; 0,631; 0,417; 1,600}. (3.34)

In ISB all elements are presented in value terms. But to complete the model development, one needs, except the vectors of intensities, the vector of prices for two subsequent years. This vector of prices was replaced by the vector of price indices. If 2008 is the base for this index, the vector of "price indices" for 2008 is the unit one:

$$\mathbf{p}(2008) = (1; 1; \ldots ; 1).$$

Now, when all the necessary data are available, Eqs. (3.31), (3.32) can be used to estimate the LTM technology matrices. Input matrix **A** is presented in Table 3.2, and the diagonal elements of the output matrix **B** are as follows:

b_{11}=131955,85; b_{22}=120564,48; b_{33}=108960,64; b_{44}=184196,31; b_{55}=102380,45; b_{66}=121649,22; b_{77}=88014,12; b_{88}=101129,02; b_{99}=84294,50;

$b_{10,10}$=76777,42; $b_{11,11}$=80871,57; $b_{12,12}$=66033,66; $b_{13,13}$=66759,56; $b_{14,14}$=71255,30; $b_{15,15}$=87664,77; $b_{16,16}$=444135,70.

The model obtained, unlike the basic LTM, is open. Additional 16-th process and 16-th goods serve here as a formal manifestation of the interaction of Ukrainian economy with other world.

The intensity of the external process is measured in amount of migrant workers, and the corresponding technological coefficients, respectively, express the costs and outputs per one such worker. Nevertheless, according to our estimations, obtained values of $a_{16,16}$ and $b_{16,16}$, have only little influence on the domestic economy and do not reflect the expences and output of the external economic system as a whole.

Table 3.2

The input matrix **A**

	1[7]	2	3	4	5	6	7	8
1	34624,57	5462,86	3355,97	9637,11	3074,59	3259,78	3193,50	5844,10
2	58,85	3703,39	63,24	62,77	48,23	55,35	47,86	133,19
3	600,90	716,89	2377,50	12892,00	16474,66	3160,51	234,30	296,58
4	31762,99	43375,79	23603,07	60519,78	25401,55	72984,02	19986,76	31775,79
5	2206,03	2840,29	6268,63	5651,68	9723,59	2713,91	1357,19	2949,30
6	5206,79	5265,22	5401,96	5338,62	5692,79	7752,29	5249,04	5484,49
7	14819,84	34563,50	4951,13	36827,04	387,49	1641,90	4312,28	2005,63
8	353,08	542,72	416,60	490,26	588,87	928,45	719,41	709,29
9	5340,72	11169,00	8713,97	9277,33	3132,96	6585,22	6995,39	2398,59
10	2319,26	3118,53	2434,61	3764,45	4178,09	3959,14	6316,65	3079,97
11	3295,66	3208,51	3181,39	4611,09	3513,69	6534,51	14055,43	8773,61
12	2411,76	2724,77	2783,08	2906,77	3495,26	2617,72	2839,20	3074,67
13	2769,94	2760,11	2793,64	2788,84	2834,61	2803,08	2835,04	2784,84
14	2013,34	1968,57	2008,80	2021,18	2036,14	2045,34	2058,48	2112,26
15	863,41	830,91	929,66	957,07	1020,54	1160,76	1797,59	2811,14
16	19877,87	31729,42	17147,19	45799,47	30827,14	40656,27	14212,66	17917,97

[7] 1 – Agriculture, hunting, forestry; 2 - Fishing; 3 - Mining; 4 - Manufacturing; 5 - Production and distribution of electricity, gas and water; 6 - Construction; 7 - Trade; repair of motor vehicles, household goods and personal items; 8 - Hotels and restaurants; 9 - Transport, storage and communication; 10 - Financial activity; 11 - Real estate, renting; 12 - State management; 13 - Education; 14 - Health care and social. assistance; 15 - Providing communities. and individual services; activities in culture and sports; 16 - Foreign production system.

Table 3.2 *(continued)*

9	10	11	12	13	14	15	16
3033,77	3002,90	3080,82	3682,44	5237,69	4591,73	3231,49	21863,13
47,73	47,73	47,93	57,24	78,59	70,40	48,63	51,48
2947,97	188,37	379,67	380,68	815,92	581,92	394,98	11452,03
27614,37	16092,53	22555,87	20139,22	19297,95	25667,42	20244,35	213199,36
4024,24	774,88	3276,12	2195,41	4970,22	4140,08	2691,95	2446,67
5372,33	5156,77	6629,64	6164,22	5433,13	5513,52	5672,69	5879,90
1424,24	1387,00	629,12	432,96	284,39	403,95	770,13	755,35
666,06	376,82	662,89	1101,16	568,46	451,19	928,16	6248,25
8143,70	2437,88	3333,14	3873,59	2155,88	2479,27	3527,40	31851,73
2930,14	9922,65	3984,96	2850,22	2377,23	2748,64	3454,30	3829,45
5104,40	5473,98	12314,44	3479,49	3466,53	3403,43	6833,04	9018,74
2664,12	2557,38	2743,06	2763,54	3523,08	2749,44	3821,45	2458,69
2843,23	2781,31	2875,93	2871,50	3125,39	2888,29	3135,95	2948,28
2095,21	2119,10	2084,99	2140,20	2055,33	2100,95	2121,15	2360,54
980,20	848,00	1858,04	1644,97	1708,17	1457,77	13834,80	2406,92
19654,12	10360,34	14013,28	12485,41	11850,84	14528,62	13653,60	8676,49

Using above technological matrices, one can try to predict, using the material balance equation (1.3), the development of processes by calculating the processes intensities for a few coming years.

But these calculations reveal that in the following years the intensity of some processes decreases and even become negative, when for others it become growing rapidly. This is due to assuming that the technology does not change for a long time and the fact that the present structure of the economic system does not allow it to get on the turnpike.

To bring the model economy on the turnpike, one needs to find basic intensities providing the balanced growth and satisfying the material balance:

$$\lambda \cdot \mathbf{A} \cdot \hat{\mathbf{y}}(0) = \mathbf{B} \cdot \hat{\mathbf{y}}(0), \tag{3.35}$$

where λ is the rate of the balanced growth, and $\hat{\mathbf{y}}(0)$ is the vector of intensities of processes for a certain base year.

It is clear that for $\hat{\mathbf{y}}(0)$ vector, satisfying (3.35), the condition of material balance will be fulfilled in the following years, as both sides of this equation will increase at λ times every year.

Expression (3.35) can be written in terms of their eigennumbers and eigenvectors:

$$\mathbf{A} \cdot \mathbf{B}^{-1} \cdot \hat{\mathbf{y}}(0) = \frac{1}{\lambda} \cdot \hat{\mathbf{y}}(0),$$

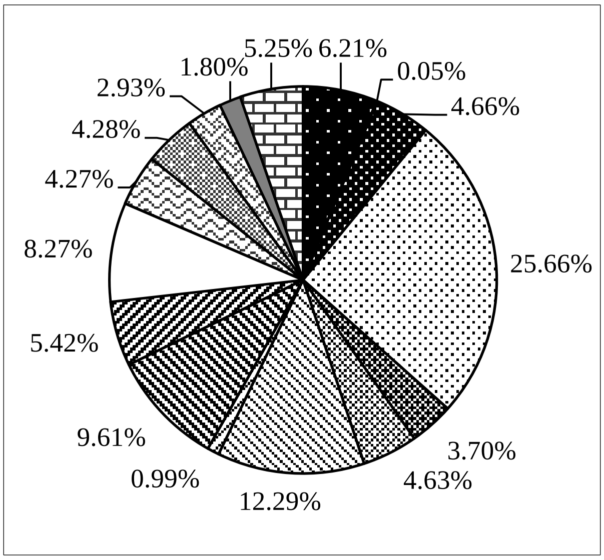
where ($1/\lambda$) is eigennumber, and $\hat{\mathbf{y}}(0)$ – eigenvector of $\mathbf{A} \cdot \mathbf{B}^{-1}$ matrix.

Eigennumbers of this matrix (as well as all other numerical results presented below) we computed using Derive 6.0 environment. The following eigennumber: (1 / λ) = 0,989 was found. From here, λ = 1,011.

Note that generally it appears not necessary to maintain the material balance for the 16-th process, because we consider the open system and external production system can provide it with additional goods.

Therefore, the last equation was removed from the system (3.35) and additional requirement of full employment was added:

$$\begin{cases} \lambda \cdot (a_{11} \cdot \hat{y}_1(0) + a_{12} \cdot \hat{y}_2(0) + \ldots + a_{1,16} \cdot \hat{y}_{16}(0)) = b_{11} \cdot \hat{y}_1(0); \\ \ldots \\ \lambda \cdot (a_{15,1} \cdot \hat{y}_1(0) + a_{15,2} \cdot \hat{y}_2(0) + \ldots + a_{15,16} \cdot \hat{y}_{16}(0)) = b_{15,15} \cdot \hat{y}_{15}(0); \\ \sum_{j=1}^{16} \hat{y}_j(0) = 22,322. \end{cases} \tag{3.36}$$



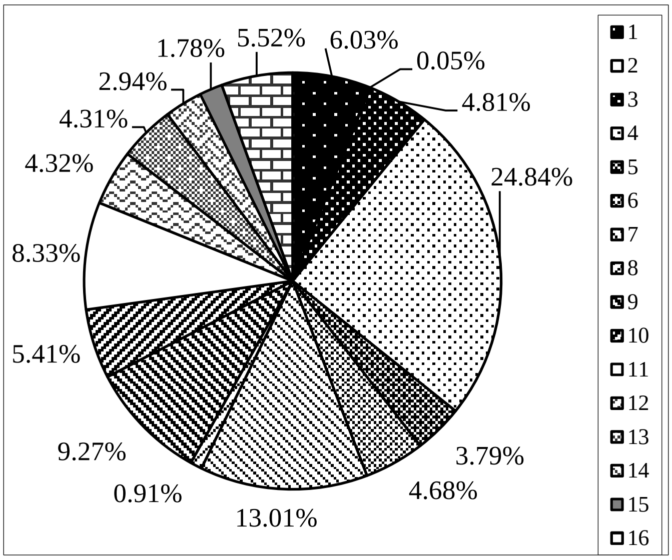


Fig.3.4. Actual (*left*) and ideal (*right*) structure of labor force distribution in Ukrainian economy.

The first 15 equations in (3.36) indicate that domestic goods are consumed by domestic processes or exported. Taking λ = 1,011, it was found the following solution of system (3.36):

$$\hat{\mathbf{y}}(0) = \{1{,}345;\ 0{,}011;\ 1{,}073;\ 5{,}546;\ 0{,}847;\ 1{,}045;\ 2{,}904;\ 0{,}204;\ 2{,}069;\ 1{,}207;\ 1{,}858;\ 0{,}965;\ 0{,}961;\ 0{,}657;\ 0{,}398;\ 1{,}232\}. \quad (3.37)$$

This solution represents such structure of processes, which allows balanced development of the open Ukrainian economy. That is, if the labor force in the initial period (2007) would be distributed according to (3.37), all processes could grow with the same rate of 1,011 maintaining financial balance for domestic goods.

The comparison of the actual (3.34) and calculated (3.37) structures of the intensity vector (in percentage of the total labor force) is shown in Fig. 3.4.

It should be noted, that comparing the actual (3.34) and calculated (3.37) intensities, we can see that employment in agriculture had to be reduced by 41 thousand people. And the trade sector, on the contrary, must be increased by about 160.9 thousand. Of course, such figures are more an effect of the poor statistics which does not always reflects the real situation. Such a sector, as trade is quite well developed in our country, but most of its real data are unavailable for statistics authorities.

Despite the fact that the system (3.36) contains material balance for the 16-th process, and satisfies the solution of Eq (3.36), which is close to the solution of (3.35). The maximum difference between them observed was eleven thousand UAH (the minor inaccuracy $9{,}37\times10^{-6}$ %) for the goods of the third sector. The above method of "optimizing" of the production processes intensities can be used for other time periods as well. In this case the growth rate remains the same, but the total number of labor force, which is available for distribution in the period under consideration is changing according to solution of (3.36).

But the pace of the real system is limited by the growth rate of labor force. If the latter is less than λ, the domestic economic system will not have enough manpower, and if more - there will appear an extra labor force. In the first case there is a need in the labor force from abroad, and the second the increased outflow of labor force abroad or growing unemployment will be observed.

Actually, the growth rate of the labor force from 2007 to 2008 in Ukraine was 1,003, which is less than λ = 1,011. This means the maximum sustainable growth is unreachable, because it would be necessary to hire additional workers from outside at a rate of 0.8% of the total labor force in 2007 (about 179 thousand people).

Thus, the method of constructing the open version of dynamic LTM, presented in [39] and based on ISB table data and illustrates on an example of our country, allows to explore its turnpike properties. The highest possible rate of sustainable growth was found and the structure of the intensity vector, allowing Ukrainian economy to develop on turnpike was found. However, it must be noted, that this method is based on the assumption of full employment, does not take into account possible technological changes and limits of the import flows.

Further study based on this model may investigate the financial balance for domestic processes and the external economic balance of whole internal system. One can also extend LTM model to ETM, taking into account the non-production sphere to learn more social components of the system and to introduce restrictions on the growth rate of the labor force explicitly.

Models based on technological matrices (LTM or ETM) have found their applications within agent-dynamic modeling. Input and output matrices restrict here the technological capabilities available for A-owner agents. They also may set restrictions for processes intensities by introducing limits for total labor force (limited number of A-worker agents, who for some reasons have no intention to work with companies belonging to certain sectors). Some of such problems are discussed in the next chapter.

3.2. Sustainability of the Ukrainian economy

The concept of sustainability is a basic one for any economic system. Even if economy reached a sufficiently high rate of growth, it does not ensure the long-term prosperity. Fluctuations in prices, the echoes of economic crisis in the neighboring countries, demographic problems, *etc*. can quickly bring it out of balance and nullify its long way of development.

In this regard, a number of questions arise. How to identify the most vulnerable, unstable elements of the economic system? What is the degree of sustainability of economy in a country? How to bring an economic system back to equilibrium?

The problem of sustainability of the economic systems has been widely discussed in the literature [37, 38], where the dynamic properties of LTM, in particular, the dynamic "input-output" model, related to LTM, were studied. However, there was paid little attention to the practical application of the results.

It would be appropriate to consider in this regard, the sustainability of a specific economy.

One of the aims of our study [39] was providing recommendations for stabilization of the economic system in Ukraine on the basis of study its dual stability property [37]. For this purpose the concept of dual stability [37] was generalized on LTM and applied in analysis of Ukrainian economy, to identify its stable and unstable components, as well as its ability to develop on von Neumann’s turnpike trajectory.

Note that sustainability of any economic system is usually studied using its models. However, one must distinguish between the model's stability and sustainability of the real system. The latter may be deduced as a consequence of the first only if the model is sufficiently adequate.

For studying the economic system of Ukraine in [39] the von Neumann’s [4] (LTM) model was used. This model is a dynamic model of a closed economy, which includies all its production processes. In practice, such a model can not be calibrated perfectly due to lack of statistical information available. Nevertheless, as it was shown in the preceeding Section, an approximate model, using input-output ISB tables can be developed [39], provided by the State Statistics Committee of Ukraine [35].

Conclusions about the LTM sustainability made by different authors in general coincide. John. D. Sargan, examining the stability of the dynamic "input-output" model [37] came to the conclusion that this model reproduces dynamic equilibrium, but it is unstable. Minor perturbations bring this model out of balance. His conclusions about the "input-output" model instability were made on the basis of estimating the eigenvalues of the core matrix of the model. Presence of positive real parts of eigenvalues found in [37] indicated instability.

Interesting are also conclusions of D. Jorgensen study [38]. He examined separately two subsystems of LTM - the material and financial. Jorgensen proved a dual stability theorem for the dynamic "input-output" model, which states that if the production subsystem is stable, the price subsystem becomes unsustainable, and vice versa.

Despite some differences between "input-output" model and LTM, which were discussed above, all these conclusions can also be applicable to LTM.

LTM for the Ukrainian economy was developed and presented in [39] (see previous Section). Each of its process is an aggregate of all the processes belonging to the same kind of economic activity, and, in fact, matches the concept of "pure industry" (each process produces only one aggregated product). Accordingly, “goods” here represent aggregates of all products manufactured by all companies included in a single aggregated process (industry).

A total of 16 processes and the corresponding products have been defined - 15 domestic and one external, reflecting the relationship with the external economic system. This makes LTM actually a model of an open economic system.

The following processes with their relevant indices were considered: 1 – Agriculture, hunting, forestry; 2 - Fishing; 3 - Mining; 4 - Manufacturing; 5 - Production and distribution of electricity, gas and water; 6 - Construction; 7 - Trade; repair of motor vehicles, household goods and personal items; 8 - Hotels and restaurants; 9 - Transport, storage and communication; 10 - Financial activity; 11 - Real estate, renting; 12 - State management; 13 - Education; 14 - Health care and social assistance; 15 - Providing communities and individual services; activities in culture and sports; 16 - Foreign production system.

LTM is defined by two technological matrices: input matrix $\mathbf{A}$ and output matrix $\mathbf{B}$, and two vectors: processes intensity vector $\mathbf{y}(t)$ and goods prices vector $\mathbf{p}(t)$.

Like the extended technological model (ETM), intensies of all processes are determined by the total number of people employed in them. Assuming full employment and equal average wages in all processes, the column vector of intensity for 2008 will have the form (3.34) (see previuos Section).

ISB tables, which are the source for the construction of LTM, were presented in the value terms. Correspondingly, the vector of "prices" turns into price indices, *i.e.*, in a set of dimensionless numbers indicating relative changes in average prices of a certain group of goods with respect to a base year. In the base year (2008), the price vector consists of units.

As mentioned above, the LTM may be unsustainable, and therefore, hardly is suitable for predicting the dynamics behavior of the real economic system. Nevertheless, an attempt was made in [39] to explore how the dynamic elements of the model (intensity vectors and prices) change, within the assumption that the equilibrium in this system is maintained in each period. For this purpose two balance equations – the material and financial (1.3) and (1.4) were used.

The equation of material balance was written as:

$$\mathbf{B} \cdot \mathbf{y}(t-1) = \mathbf{A} \cdot \mathbf{y}(t) .$$

From the material balance the vector of intensities of the next year can be expressed as follows:

$$\mathbf{y}(t) = \mathbf{A}^{-1} \cdot \mathbf{B} \cdot \mathbf{y}(t-1) . \qquad (3.38)$$

It turns out that the intensities of some process grow quickly, when intensities of others decrease and even become negative. In the next periods the situation changes. That is, the system performs the increasing intensity of the cyclical fluctuations, as was expected and illustrated in Fig. 3.5.

The dynamics of the processes intensity in logarithmic scale ($sign(y_j(t) \cdot \ln(abs(y_j(t)))$, j = 1,…,16.) is illustarated In Fig. 3.5. It is clearly seen that the system is unable maintain balance, it does not tend to any stationary state.

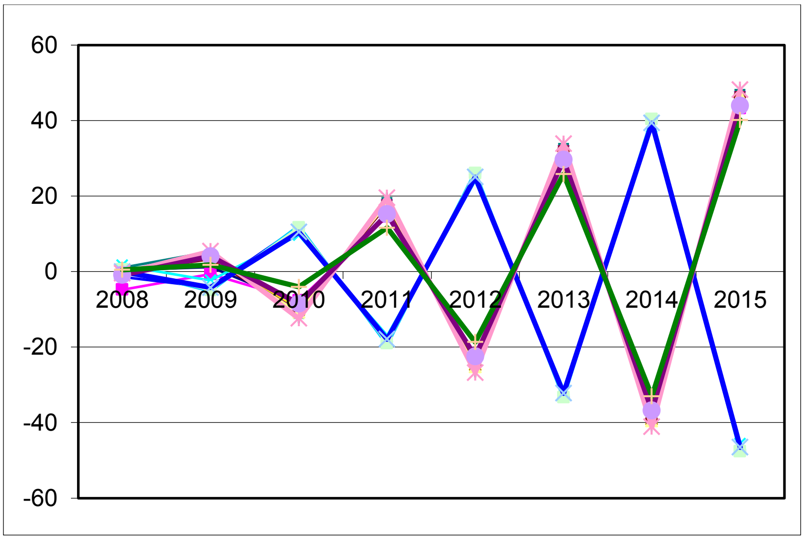


Fig. 3.5. Instability of the intensities

Financical balance is written on the assumption of a perfect market, in which any agent can get profit. At financial equilibrium expences of the previous period of each process are reimbursed by current income, *i.e.*:

$$\mathbf{p}(t-1)\cdot\mathbf{A}=\mathbf{p}(t)\cdot\mathbf{B}\,.$$

From the financial balance equation the vector of prices can be expressed as follows:

$$\mathbf{p}(t)=\mathbf{p}(t-1)\cdot\mathbf{A}\cdot\mathbf{B}^{-1}.$$

Similarly, the dynamics of price indices can be calculated starting from a known initial vector. However, for the prices system the situation is different. Indices of prices fluctuate slightly at first, but in contrast to the case of the prosesses intensities the oscillation of prices, as one can see in Fig 3.6, are damped. After a few periods all the price indices begin to decrease at the same rate. Thus, the LTM system of prices for the Ukrainian economy is sustainable; it tends to a certain stationary state, which also satisfies the condition of the financial equilibrium. It should be noted that the LTM does not accounts for additional emissions and inflation.

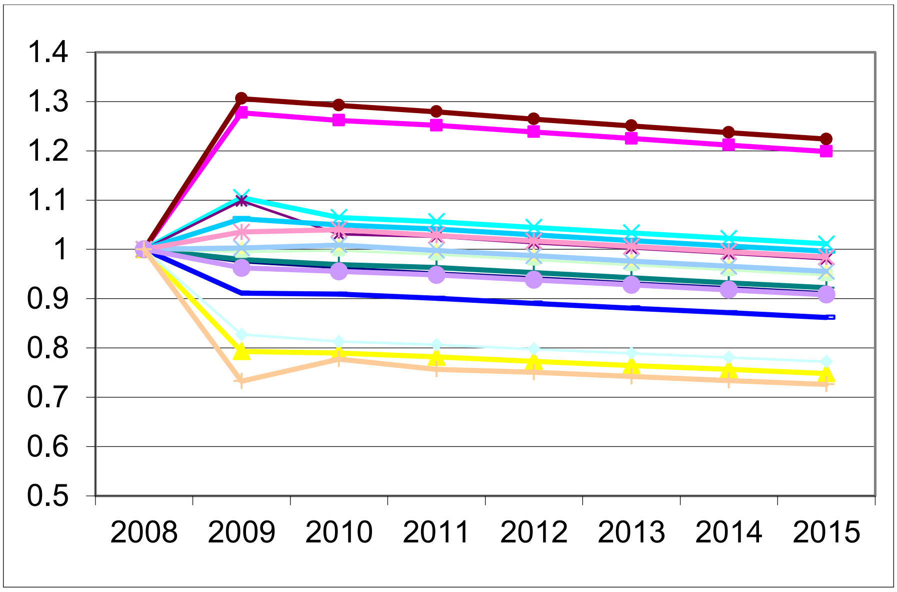


Fig. 3.6. Sustainability of the prices vector

As it is can be seen from Fig. 3.6 the prices indices change considerably: some price should have been increased, on other - to decrease. The price vector should reach a steady dynamics in a few years, and in 2012 has to be as follows:

p(2012) = {0,943; 1,238; 0,773; 1,045; 1,013; 1,264; 0,953; 0,891; 1,029; 0,798; 0,979; 0,989; 0,987; 1,017; 0,938; 0,751}.

Prices predicted by this model, reflect those changes in the real prices which should be made to achieve sustainable economic dynamics. For example, the prices of the 6-th industry products (construction), according to this idealized model have to grow 1.26 times compared to 2008, and the prices of mining products (3-d industry) must decrease, reaching only 77% of those at baseline.

In any case, because the material production in any industry is the primary issue, its instability in the considered equilibrium models should be eliminated. Such possibility was also discussed by Jorgensen [38], who noted that both subsystems can be stabilized if the interest rate is equal to the rate of economic growth.

At the unique trajectory of development, where all the processes are growing and prices of all goods are reduced at the same rate, known as the von Neumann's turnpike:

$$\mathbf{y}(t) = \lambda \cdot \mathbf{y}(t-1) \text{ and } \mathbf{p}(t) = \mathbf{p}(t-1)/\lambda . \tag{3.39}$$

Here λ – is the maximum rate of balanced growth.

To make the entire system developing on the turnpike, the initinal state must belong to it. Since the price system will become on to the turnpike on its own, we need to adjust only the structure of the intensity vector. At the same time λ can be calculated from the resulting dynamics of the price indices.

Since prices will change in a balanced manner, it is possible to follow any product. For example, using p_{14} (2014) = 0.995, and p_{14} (2013) = 1.006 follows λ = 1,011.

To find the initial vector of intensities belonging to a turnpike, it is necessary to rewrite the material balance (3.38) with account of (3.39) in terms of eigenvalues and eigenvectors:

$$\mathbf{A} \cdot \mathbf{B}^{-1} \cdot \hat{\mathbf{y}}(0) = \frac{1}{\lambda} \cdot \hat{\mathbf{y}}(0) .$$

This problem was solved in [39]. The growth rate obtained was: λ = 1,011, *i.e.*, the same as obtained using method of financial balance [36]. This confirms the correctness of above calculations, and the dual nature of LTM stability.

The vector of the intensities for 2007, which would make it possible to develop on turnpike, was also estimated in [39]. For 2012, taking into account the labor force 21930 thousand people (forecasted in [39] according to [35]), this vector would look like this (in thousands of people):

$\hat{\mathbf{y}}(2012)$ = {1,322; 0,011; 1,054; 5,448; 0,832; 1,026; 2,853; 0,200; 2,033;1,186;1,826; 0,948; 0,944; 0,645; 0,391; 1,210}.

This vector reflects the structure of employment at various processes, which provides proportional consumption and production of goods taking into account the inter-sectoral links. Maintaining such a structure of the production system must provide equilibrium economic growth, which is one of the main objectives of the government economic policy.

The study of such properties of the LTM as dual sustainability carried out above on the example of the Ukrainian economy, revealed that the price system within this model is stable, and the intensities of the system processes are not. As it was shown in [39], stabilization of the economy is possible by bringing the scale of production of various branches of economy to the calculated ideal intensities.

However, in practice this is not always possible, firstly due to limitations of the economic policy, and secondly because of the various restrictions (natural and human resources, international relations, etc.) not accounted within LTM. Another drawback of this model is the assumption of the absence of technological progress (the elements of technological matrices assumed to be constant).

Many of these limitations can be eliminated within the same model by extending it, *i.e.* adding additional products and processes [25], as described in Section 2.2.

In conclusion we note that LTM reflects only technological side of the problem and does not take into account the motives of the subjects of economic activity. The market price will not necessarily be established according to the above "optimal" scheme. The same is valid for resources - they cannot be distributed as is better to all people. Therefore, to present a better solution, it is necessary to combine to the LTM (or ETM) with agent-based modeling, *e.g.* with ADM approach. This may allow including the interaction of individual economic agents, both within an industry and at the national (international) level.

3.3. *Optimization of the export and import flows*

One of the main problems of Ukrainian economy is its extreme dependence on imports. For example, in 2010 the export-import deficit amounted to -31.579 billion UAH [35]. Such a deficit threatens the economic security of the state, and increases its foreign debts.

On the other hand, reducing imports, especially raw materials and energy could lead to decrease of production, rising unemployment and so on. Below we discuss how the optimal flows of exports and imports using various criteria [40] can be found using the extended linear technological model (ETM) of an open Ukrainian economy (see Section 2.2) calibrated using input-output ISB tables, as described in Section 3.2.

As it was stated above, the ETM model uses two matrices: input matrix **A** and output matrix **B**, as well as two vectors: vector of processes intensity $\mathbf{y}(t)$ and vector of goods prices $\mathbf{p}(t)$.

Model considers the existence of 15 national processes (see Tabl.3.2) and related products and one aggregated process reflecting the flow of imported goods. It represents the external (abroad) economic system (index 16).

The output of the external process is measured in amount of number of workers, producing imported goods, and the corresponding elements of technological matrices express, respectively, the input and output per one such worker. Intensities in such system are unstable, and the actual intensity vector of the Ukrainian economy is not approaching to the turnpike. In the previous Section it was shown how it is possible to change the structure of the employment to bring economy on turnpike [39].

In general case, this is very difficult problem due to limited possibilities of the labor force re-distribution between the processes and need of balancing exports and imports. Below we consider only one of these problems, limiting the balanced growth of Ukrainian economy, namely, how can be corrected the parameters of the export and import flows.

It is assumed that the current structure of the labor force for all processes is constant for a long enough time. Technological matrices and production costs in the basic model considered as stable. This condition may also be applied to factors that govern domestic consumption and production processes, if changes in technology are not considered.

If the technology be stable and imported goods cannot be replaced by domestic ones, then $a_{ij} = const$ (i=1,.,16, j=1,…,15). Element $a_{16,16}$ is a property of the abroad system and is not important in this problems.

The elements $a_{i,16}$, (i =1,.., 15) represent the consumption of domestic good i by an external system per unit of its intensity, may vary. We denote them as export coefficients: $a_{i,16}=ex_i$.

Let's consider an economic system with sustainable balanced growth:

$$\mathbf{y}(t+1) = \lambda \cdot \mathbf{y}(t)\,, \tag{3.40}$$

where λ is the rate of economic growth.

Within this model only we require for material balance only for domestic products (because most of the imported goods manufactured abroad and not reflected in the model).

Material balance for domestic goods, with account of (3.40), gives the following set of equations for distribution of labor force in the base period **y**(0):

$$\begin{cases} \lambda \cdot (a_{11} \cdot y_1(0) + a_{12} \cdot y_2(0) + ... + ex_1 \cdot y_{16}(0)) = b_{11} \cdot y_1(0); \\ \lambda \cdot (a_{21} \cdot y_1(0) + a_{22} \cdot y_2(0) + ... + ex_2 \cdot y_{16}(0)) = b_{22} \cdot y_2(0); \\ ... \\ \lambda \cdot (a_{15,1} \cdot y_1(0) + a_{15,2} \cdot y_2(0) + ... + ex_{15} \cdot y_{16}(0)) = b_{15,15} \cdot y_{15}(0). \end{cases} \tag{3.41}$$

Equations (3.41) mean that everything produced in the country is consumed next year in domestic industries or exported.

We can consider two tasks related to regulating the export flows.

1) Finding the export ratios at which existing distribution system of labor force can develop balanced and provide full employment.

2) Finding the import-export ratios and growth rate of the economy leading to the zero export-import balance.

Considering the first task, we can mention that the rate λ of balanced growth should be equal to the growth rate of the labor force. In general, it changes over time.

The system (3.41) consists of 15 equations and 15 unknown export ratios, and is solution can be expressed as follows:

$$ex_i = \frac{1}{y_{16}(0)} \cdot \left(\frac{1}{\lambda} \cdot b_{i,i} \cdot y_i(0) - \sum_{j=1}^{15} a_{ij} \cdot y_j(0) \right),\ i=1,\ldots,15. \quad (3.42)$$

The rate of sustainable growth of labor force can be regarded either as a constant, or as a variable (estimated) value.

Export coefficients (3.42) at the rate constant λ = 1,00336, which was estimated from the statistics data [35] for 2008, technological matrices and vector of intensities for year 2008 [39], were obtained from the one (most important in this case) seventh process ("Trade and repair"). It was found that Ukraine lacks trading services even to meet their own needs. We regarded it as a consequence of poor statistics, because one can hardly speak about the lack of trading services in Ukraine.

To make the zero export ex_7 in this problem we increased the output rate for the seventh process up to b_{77} = 94524,73.

According to equation (3.42) and to the above value of b_{77} we calculated vector of export coefficients for sustainable development at a constant rate of population growth. It is shown in the fourth column of Table 3.3. This vector represents the size of the export of certain goods per worker, necessary for fulfillment of the full employment condition.

Table 3.3

Export coefficients: actual and calculated, in UAH per employee

Process No.	Short name of the process	Actual	1) Full employment	2) zero balance
1	Agriculture	21863,13	26371,12	34002,95
2	Fishing	51,48	41,29	95,59
3	Mining industry	11452,03	7930,69	12660,88
4	Manufacturing	213199,36	251462,08	295498,07
5	Electricity, gas and water	2446,67	635,34	4160,25
6	Construction	5879,90	5514,67	10757,82
7	Trade and reparation	755,35	0,00	0,00
8	Hotels and restaurants	6248,25	8175,66	9108,07
9	Transport and communications	31851,73	39294,88	46843,76
10	Finance	3829,45	4960,09	8834,42
11	Real estate	9018,74	10588,14	16818,72
12	Governance	2458,69	2243,63	4873,82
13	Education	2948,28	2994,70	5655,16
14	Health	2360,54	2488,11	4432,79
15	Community services	2406,92	3057,02	4526,98

But within such problem the requirement of zero export-import balance, which is important for normal relations with its neighbors and economic independence, is not considered.

The volume of imports should be compatible with the volume of exports. If a country has nothing to offer instead the goods received from abroad, the external debt begins to increase. Realization of export goods normally provides the currency needed for imports' payments. Moreover, these payments could be done within "year by year" scheme, with some time lag, which must be is fulfilled in the material balance equation (3.41).

In terms of current model, the technological matrices elements are presented in the value terms, and it is believed that the exchange rate did not change significantly during one year, therefore the total imports in period *t* equals to $\sum_{j=1}^{15} a_{16,j} \cdot y_j(t)$, and exports to $\sum_{i=1}^{15} ex_i(t) \cdot y_{16}(t)$.

As a result, optimizing the export flows from the standpoint of the full employment for 2008 it was obtained [39] the negative export-import balance of -108,8 billion USD. Actually the balance in the Ukraine was only -75.729 billion UAH. [35]. That is, when trying to achieve the full employment of the labor force, Ukraine can significantly worsen its external economic situation.

Therefore it makes sense to consider the second task mentioned above, *i.e.* find such import-export ratios and growth rate of the economy which correspond to the zero export-import balance.

That means the restriction:

$$\sum_{j=1}^{15} a_{16,j} \cdot y_j(0) = \sum_{i=1}^{15} ex_i(0) \cdot y_{16}(0) \tag{3.43}$$

has to be added to set of equations (3.41). Now the system (3.41), (3.43) contains 16 equations and 16 unknowns, including λ. After substituting (3.42) into (3.43), the solution of (3.41) gives the balanced growth rate of:

$$\lambda = \frac{\sum_{i=1}^{15} b_{ii} \cdot y_i(0)}{\sum_{i=1}^{16} \sum_{j=1}^{15} a_{ij} \cdot y_j(0)}. \tag{3.44}$$

Thus, the rate of balanced growth needed to keep foreign trade balance (3.43) equals to the share of production of all domestic processes to its consumption.

This is analogous to the maximum possible rate of of sustainable growth found by von Neumann for closed economy [4].

For the model under considerstion according to (3.44) we obtained λ = 0,9564. That means, when keeping zero foreign trade balance and not to borrow, technologies available in Ukraine will lead to annual recession of 4.4% instead of economic growth. This in turn, will increase the unemployment and gradually deteriorate the economic situation, because the average rate of drop in labor force is lower (0.997 for 2001-2009 [35]).

Substitution of λ from (3.44) into (3.41) gives the corresponding export coefficients (see Table 3.3). It shold be noted, that for the "Trade and repair" process the export coefficient become negative and the corresponding element of the output matrix has been adjusted (b_{77} = 90101,57).

The comparison of the calculated and actual export coefficients for both cases: 1) with the full employment and 2) with zero export-import balance is presented in Fig.3.7. This chart displays the ratios of the calculated coefficients to their actual values.

For example, for the fifth process (production and distribution of electricity, gas and water), exports in this sector must be almost 75% reduced, to fulfill the requirement of the full employment and to increased by 70% under requirement of the zero export- import balance.

That means solving one of these problems makes it impossible to solve the other.

It should be emphasized than Ukrainian domestic production system is too dependent on imports, so achieving of full employment increases the negative export-import balance. And when the problem of minimizing the external debt is solved, the problem of economic recession and, consequently, of unemployment increase arises.

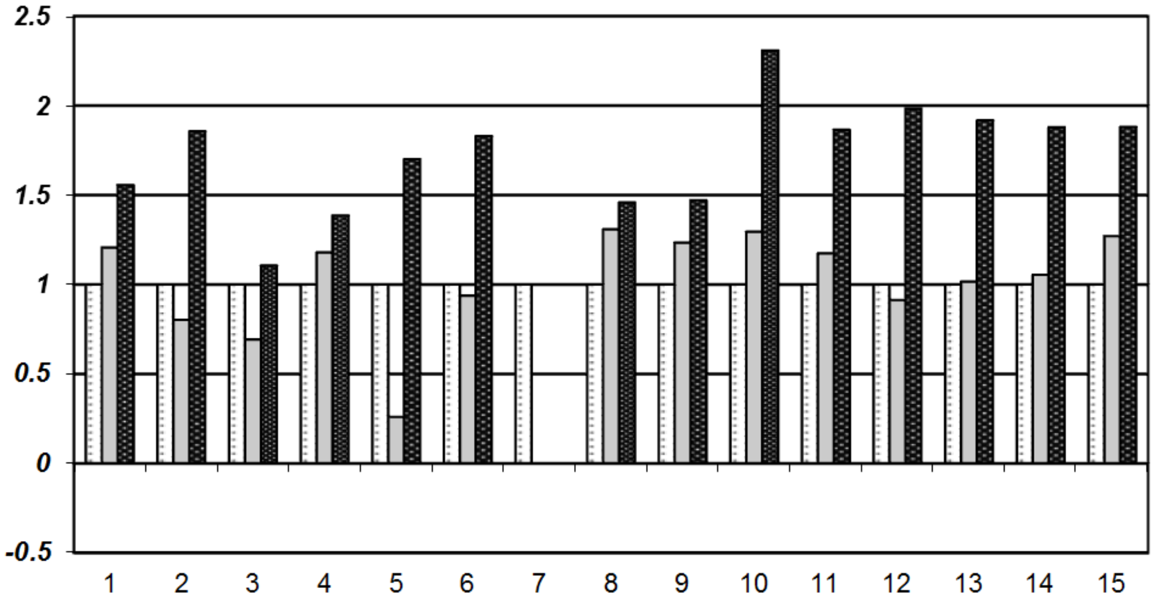


Fig.3.7. Comparison of existing export flows and solutions of two optimization problems

(white – Actuall, Gray–Full employement case, Black – Zero balance case)

Therefore, balancing of exports and imports under conditions of existing production technologies leads to a cessation of the economic growth and recession. That means the existing production technologies are inefficient.

The situation when the imported goods can not be replaced by domestic products in reality is not always the case. Most of the imports have domestic analogs which could be produced by domestic industries.

Therefore, we need an advanced formulation of the problem making some imports coefficients flexible. But it needs to know exactly which imported goods are consumed. In the technological matrices [39] all the imports are aggregated into one. It is desirable to introduce import in the expanded form.

At one of the stages of the technological matrices construction (see. previous Section) the ISB with expanded imports was introduced [39]. The entire set of existing products was represented both by domestic and similar imported products. Similar products, both imported and domestic, are considered below as interchangeable and indicated within the model by indices i and (i + 15), respectively (i=1,…,15).

Domestic products are considered as produced by relevant domestic processes which are indexed by j = 1,.., 15, and all imported as manufactured by one aggregated external process (index 16).

Technological matrices are built the same way as it was described in [39], without aggregation of imported goods. Matrix will take a notable block form as in [24]. Schematically the obtained matrices can be represented as follows:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_D & \mathbf{A}_{ex} \\ \mathbf{A}_{im} & \mathbf{A}_F \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} \mathbf{B}_D & 0 \\ 0 & \mathbf{B}_F \end{pmatrix}, \tag{3.45}$$

where $\mathbf{A}_D$ is (15×15) input matrix for domestic processes consuming domestic products;

$\mathbf{A}_F$ is input matrix of foreign processes consuming imported goods (in this model its details are not important);

$\mathbf{A}_{ex}$ is the vector of export coefficients ex_i i=1,…,15;

$\mathbf{A}_{im}$ is (15×15) input matrix for domestic processes consuming imported products;

$\mathbf{B}_D$ is (15×15) output matrix for domestic processes;

$\mathbf{B}_F$ is the output matrix of foreign processes (unimportant within this model).

For the model under consideration was obtained technological matrix (3.45) for the Ukrainian economy. Within this model, for example, it is possible to find out how the economy will develop in the limiting case when imports and exports are absent. In this case economy must be regardes as a closed and all imported goods must be replaced by domestic goods produced by relevant domestic processes.

Model of the closed economy includes 15 processes and 15 goods and the elements of input matrix are calculated from (3.45) as sums of the corresponding coefficients for similar imported and domestic products:

$$\mathbf{A}_Z = \mathbf{A}_D + \mathbf{A}_{im}, \text{ or } az_{ij} = a_{ij} + a_{i+15,j}, \; i, j = 1,\ldots,15. \qquad (3.46)$$

The input matrix for a closed economy was built according to equation (3.46), which reflects the need for goods regardless of their place of production and the output matrix equals to $\mathbf{B}_D$ from (3.45).

To investigate such a closed system, it was supposed that the domestic labor force engaged in production of export goods, become unemployed and $\mathbf{y}_i$ (0) (i = 1,.., 15) remain the same as in open economy.

The highest possible rate of sustainable economic growth for this closed system was estimated to be λ = 0,962. More specifically, it means that the von Neumann's turnpike corresponds to economic recession.

This indicates not only the inefficiency, but also high dependence on imports, and lack of self-sufficiency of Ukrainian economic system. In the best case (with no foreign relations) it could be obtained the minimal annual economic downturn of 3.8%.

Of course, the closed model in this case is built with a high degree of abstraction. In today's economy many processes use goods that are not produced in the same country and which can not be replaced by other domestic products. Therefore, at the existing technology the cost of imported goods will remain the part of the processes cost. It is possible only to adjust to a certain extent these costs on imports.

It can be done using computer simulation. Varying certain elements reflecting imports one can track the change of the maximum possible growth rate and compare it with the rate of growth of labor force, etc.

Elements of input matrices in the model with expanded imports in this case are variable. However, as the technology itself does not change, the equality

(3.46) must be maintained. Matrix elements of the $\mathbf{A}_D$ input matrix are written according to:

$$\hat{a}_{ij} = az_{ij} - \hat{a}_{i+15,j}, \; i, j = 1,\ldots,15,$$

which follows from (3.46), and $\hat{a}_{i+15,j}$ are the matrix elements to be adjusted.

Elements of the input matrices corresponding to external processes ($\mathbf{A}_{ex}$, $\mathbf{A}_F$), remain unchanged. And the import elements ($\mathbf{A}_{im}$) can vary from zero up to the actual value ($\hat{a}_{ij} \in [0, a_{ij}]$, $i = 16,\ldots,30$, $j = 1,\ldots,15$).

Thus reducing imports, the researcher can follow the changes in λ. However, because the expanded technological matrices of the model are not square, to find λ after every changing of import coefficients he has to aggregate the imports again. To avoid this, instead of the expanded output matrix an output matrix B from [39] can be used.

Using this aggregated matrix, the value of λ is calculated from the simulation of the financial balance (based on the dual stability theorem [38]). It is taken equal to the reverse of the rate of prices growth at approximately the 15-th year of modeling when it goes to be a constant. The more it is, the better. Experiments with the model show that the decrease of imports usually leads to a decrease in λ. However, this decrease is not very large, and agrees with the rate of the labor force growth.

Due to the fact that the growth rate of labor force is less than the maximum possible economic growth, which has been obtained, the lower limit for λ may be set to the average predicted growth rate of the labor force: $\hat{\lambda} \geq \overline{l}_n$.

The average predicted growth rate of the labor force can be obtained from the working age population in the given time period. For example, one can use the age shift method. According to this forecast to 2015 without considering migration $\overline{l}_n$ = 0,994 was obtained.

Another example of application of this model is its use for regulation of import flows. For Ukraine, which has the entrenched status of agricultural country, one can assume that it will be satisfied by its own agricultural production. Therefore the import elements for the 16-th goods can be set to zero.

Also, practically to zero can be lowered the fish farming import and the import of services (goods with indices from 20 to 30).

Another case are the goods 18 and 19 (products of mining and manufacturing). Exactly these categories of goods are imported in large quantities, and many of them cannot be replaced by domestic counterparts. Therefore for these goods imports decrease not too much as long as the maximum possible rate of economic growth does not approach to $\overline{l}_n$.

Performing such correction of import flows it one should keep in mind that in order to allow the system with the resulting matrix to get on to the turnpike, it also needed to adjust the initial intensity of processes (employment) as it was made in [39]. The resulting matrix can also be the subject of further correction of export flows as shown above. If negative values are obtained, the imports have to be increased.

4. Multi-agent modeling of socio-economic systems

4.1. Implementation of ADM within the AnyLogic environment

As it was stated above, agent-dynamic modeling (ADM) allows building hybrid models, which preserve advantages, and at the same time eliminate the disadvantages of the known classical economic models [5]. As a consequence, ADM can be applied in solution of a number of real economic problems. Development of ADM-applications require computer simulation environment which supports both the agent-based modeling and allows implementation of extended linear and non-linear technological models, discussed above.

The first version of such environment, the ADM "ECO-Dynamics» system, was developed [19] as Windows-based console application using C++ language. Its main aim was to demonstrate benefits of combining the ABM with theoretical models of the economy. The first step in this direction was inclusion of only one type of agents, A-Owners, who control production processes in simulation of microeconomic dynamics within LTM, keeping financial balance for each running process and maintaining the profitability of each product produced by changing σ and ε parameters of the model described above in Section 2.3.

The next step in development of ADM-supporting system was porting of "ECO-Dynamics» to AnyLogic environment, which has a rich toolkit, useful for the future development of ADM-simulation models.

Below we present some results of testing this system on a problem of the investment projects implementation by A-Owners agents, who take decisions managing the von Neumann's processes.

One of the most significant shortcomings of the von Neumann model is that it is actually built for a planned economy, and it does not take into account the limits of resources (production capacity, human resources, market capacity, *etc*.) existing in a real market economy.

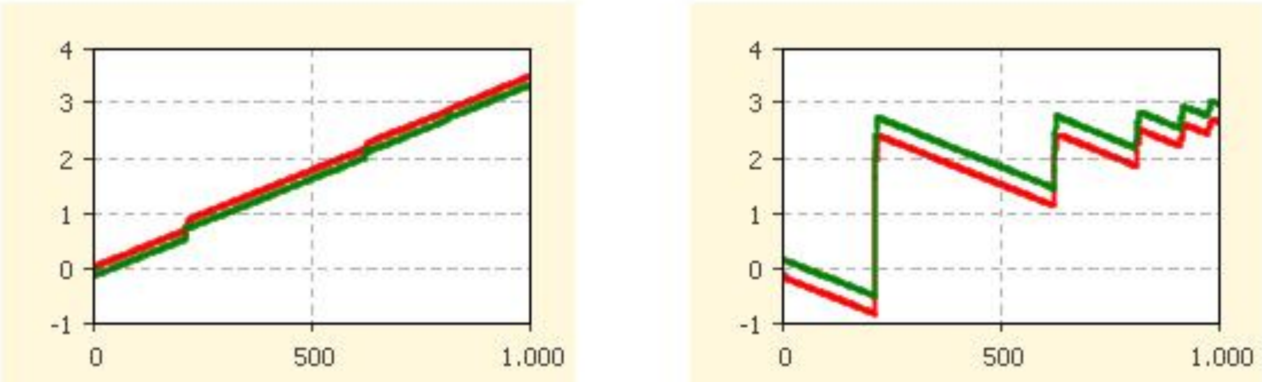


Fig. 4.1. Dynamics of intensities and prices (1^{st} experiment)

Within the concept of von Neumann model all processes have infinite capacity, fixed assets do not wear out and the fixed capital goods are free. Economic growth within the framework of this model is accompanied by continuous fall in prices on all products. In order to correct these shortcoming, two-sector non-linear NTM (see Section 2.5) was developed, which takes into account that the actual intensity of each process is limited to a certain upper bound of intensity, and its additional increase requires investments to the fixed assets (productive accumulation). This, in turn, requires credits, increasing the production costs, and this eventually is reflected in prices on products manufactured.

So, in ADM-version of ETM the intensity of each process were considered to be limited to some marginal intensity x_j^{max}. With increasing intensity of *j*-th process, and approaching it to this maximum intensity, A-agent (owner of the process) takes (or not, depending on its marketing strategy) a decision to invest in order to increase productivity beyond the current intensity limit.

In this case the intensity of the process continues (or not) to grow, but the costs associated with the investment, lead (or not) to an additional increase of the price on the manufactured products.

Fig. 4.1 presents (in a semi-logarithmic scale) the results of the first simulation using the above two-sector model.

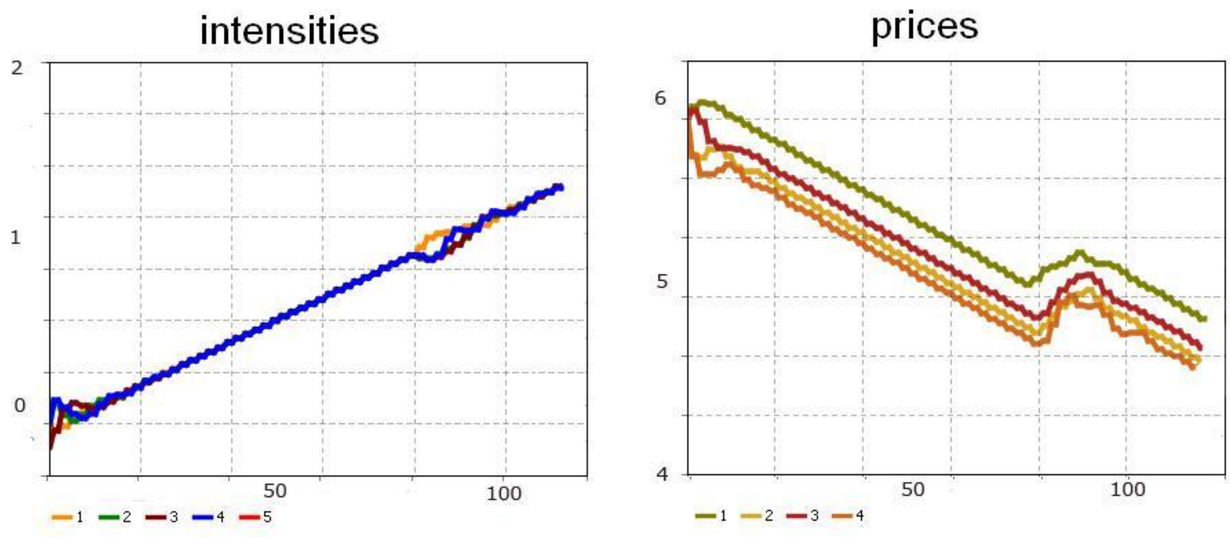


Fig. 4.2. Dynamics of intensities and prices (2^d experiment)

The system modelled includes one production and one non-production process. This model allows for the reproduction of the labor force in the non-production sphere, considering them as a specific "product" created by this process, and therefore, the possibility to introduce the non-productive consumption into the von Neumann's model.

Production capacities of the production processes are considered to be limited. As soon as the intensity of the production process reaches its maximum, the A-owner of the production process takes a decision to invest in order to expand the productivity.

In terms of the financial balance of each process, this leads to an increase in the prices of manufactured products and, because of the requirement to ensure the effective demand, to increase the labor force prices, *i.e.* wages.

In Fig. 4.2 in semi-logarithmic scale are presented the results of the second simulation of another economic system consisting of five processes, producing four products consumed by other processes. The initial dynamics of the system corresponds to the von Neumann turnpike.

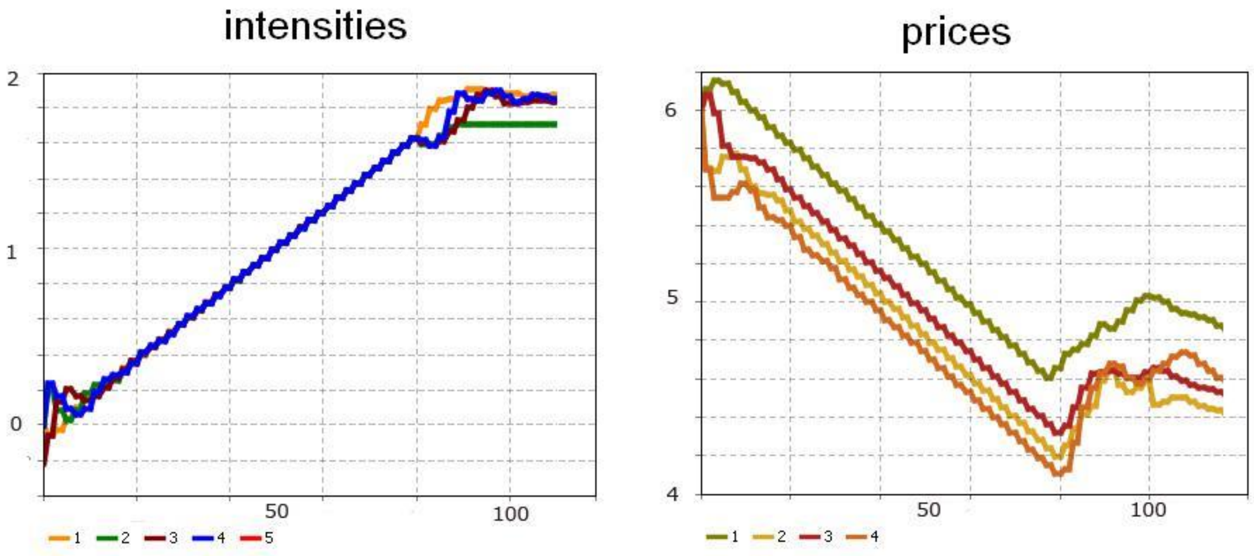


Fig.4.3. Dynamics of intensities and prices (3[d] experiment)

After reaching the limit of his process intensity, A-Owner of the process I decided to raise this limit and takes a loan. Necessity of its repayment has led to an increase in the price of his product, which due to the financial balance of each process immediately affected the prices of all other products.

In Fig. 4.3 are shown results of the thirs computer simulation of the same system, but under conditions where only one A-Owner decided to take a loan in order to increase the the limiting intensity $x_1^{\max}$. The rest of the A-Owner agents, although have reached the limit the intensity, have decided not to raise it. As a result, intensities of the processes controlled by these four agents has stopped at nearby $x_j^{\max}$, $j = 2,3,4,5$.

No substantial increase in the intensity of the first process happens despite the loan, taken by the first A-Owner agent because the value of goods released by the first process in the current period depends on the release of the other processes in previous period.

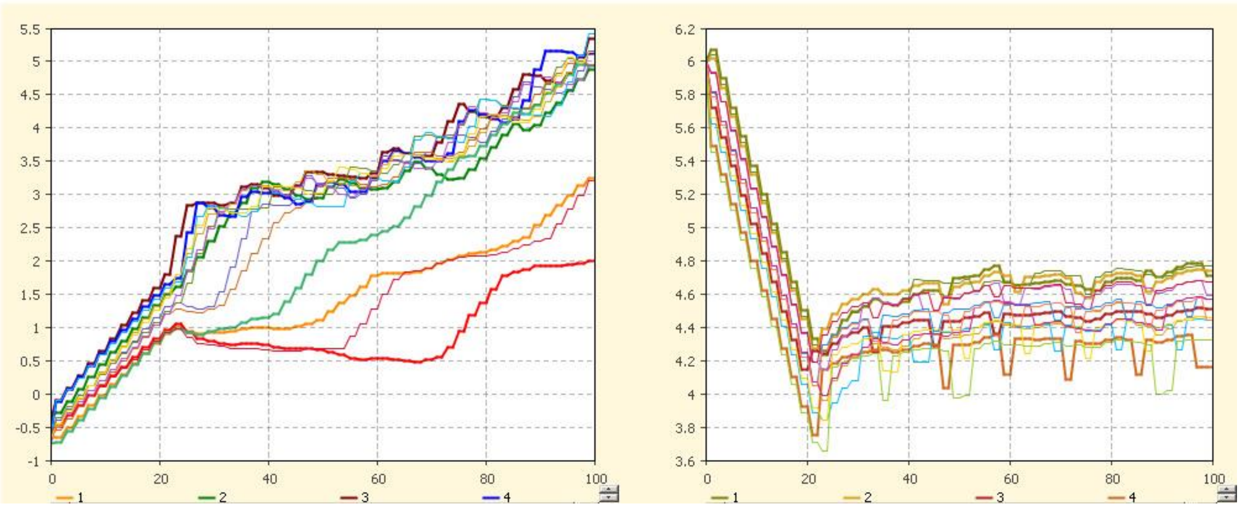


Fig.4.4. Dynamics of intensities and prices (14 processes)

Fig. 4.4 presents[8] results of a simulation, where 14 production processes, producing 14 kinds of products are interacting within NTM. The initial dynamics of the system corresponds to the von Neumann model turnpike. After reaching the limit of intensity, the agent - owner of one of the processes makes the decision to increase the limit of the intensity of its process. To extend his capital assets, he takes a loan. The need for loan repayment leads to an increase in the price of his product, what affect the financial balance of each process, and the prices of all other products respond immediately. It should be noted that the intensities of processes continue to grow over time, albeit at different rates, and prices stabilize.

Thus, using the ADM approach within NTM it becomes possible to overcome some of the shortcomings of the original von Neumann theoretical model of an expanding economy and apply its extended version in development of hybrid ADM models of real economic systems.

[8] The curves are also plotted in semi-logarithmic scale.

4.2. The impact of the mentality factors on economic growth

Differences in behavior stereotypes of economic agents in different countries, which are generally indicated by the term "economic" [41] or "law" [42] mentality, have long been of interest in economic science, as well as in the bordering areas of social psychology. The impact of cultural norms on economic behavior is reflected in the typology proposed in Holla [43], Lyuisa [44], and Hofstede [45]. The economic problems of the Russian mentality and the main factors of its formation have been extensively studied by Nureyev [42], Latov [47], Shastitko [48], and Murunova [49].

A number of publications of Ukrainian scientists reflecting the growing interest to the problems of the national economic mentality appeared in recent years. In the paper of Galushka and Luste [41] the correlation and regression analysis was applied to study how the mental characteristics of the society affect the economic development indicators of the country. In the work of Ossietzky [50] the economic mentality is considered as a factor influencing the institutional environment. It was found that the differences in traditional models of economic behavior of A-owners and wage earners, in the priorities of their economic objectives, may have a significant impact on the potential growth rate of the economy.

As a result of Hofstede's study [45] 116 thousand of IBM employees from 40 countries were interviewed. According to his typology, the most significant characteristics of the economic mentality are 1) Individualism, 2) Power Distance, 3) Masculinity, 4) Uncertainty Avoidance, and 5) Long Term Orientation. All figures of Hofstede usually vary in the range of 0 to 100 points. The "individualism - collectivism" describes the degree to which staff prefers to act independently, achieving individual goals. Power Distance determines the extent to which employees perceive differences in the status of individuals. As a “Masculine” Hofstede called society with a strong separation of social roles and job functions between the sexes. “Uncertainty Avoidance” describes how uncomfortable feel member of the society at conditions of instability. High values of “Long Term Orientation” means the predominance of the thrift in the society, as a way to achieve positive changes in the future.

The typology of organizational cultures Hofstede [45] can be taken as a basis for assessment of the economic mentality impact on the rate of economic growth.

The two most important, from our point of view, factors, whose role can be studied in the framework of ADM, were analysed below:

1. ***Power Distance***. This parameter describes whether the company income inequality is perceived as a norm. We assume that in a society with a higher value of “Power Distance” the entrepreneur will be more likely to direct a significant portion of the company income on his personal needs. The assumption was based on the fact that among the leaders of the countries on the capital runaway to offshore the value of “Power Distance” is usually quite high: in China - 80, Brazil - 69, Kuwait - 90 (for comparison, the US only 40). In his work [45], Hofstede also notices that the typology of the high level of corruption and income inequality in societies with a high «Power Distance” [45]. That mental characteristics will be referred below as "rationality" (rat).
2. ***Long Term Orientation***. The highest values of this parameter correspond to countries that have made an "economic miracle" in the 50^s of the 20^{th} century: Germany - 83 Japan - 88 (for comparison, US - 26). In Japan, in the period between 1948 and 1972 production *per capita* grew by 8.2%, and in (West) Germany - by 5.7% per year. At the same time in the US the growth rate was only 2.2% [51].

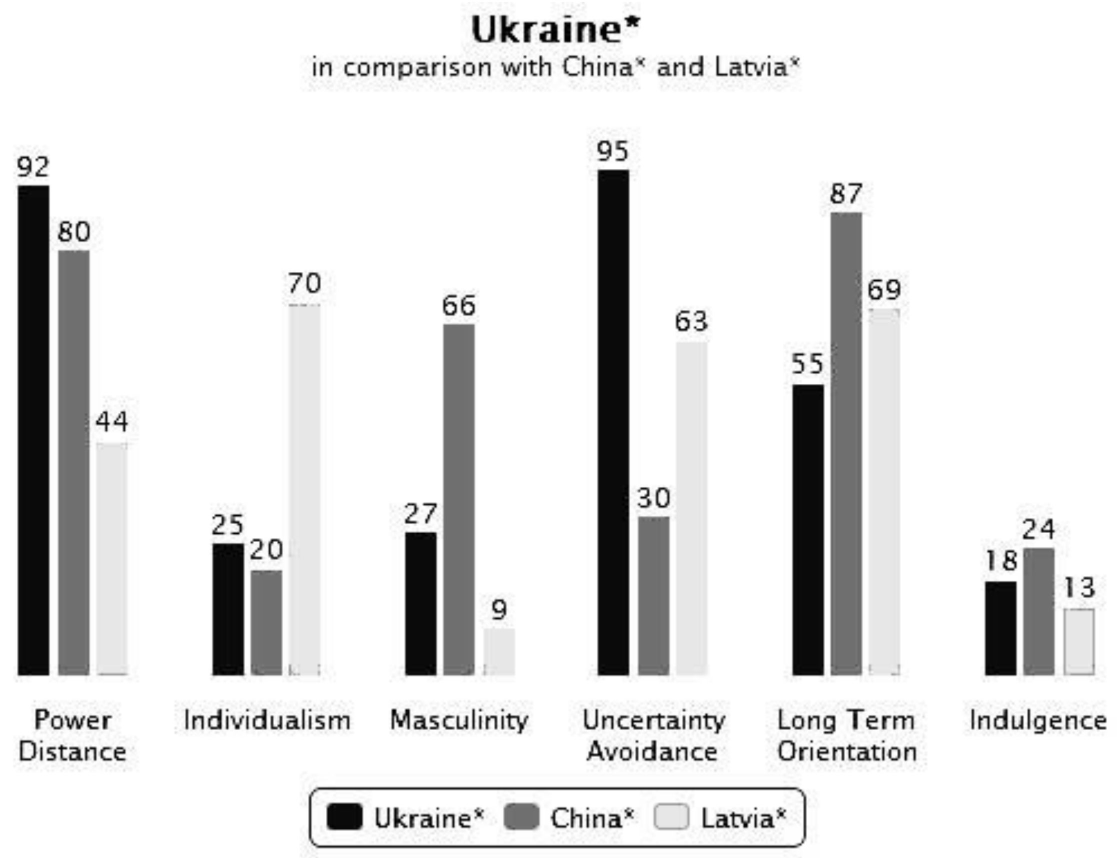


Fig.4.5. Comparative analysis of the characteristics of the economic mentality in the Ukraine, China and Latvia in terms of the Hofstede typology [52].

In the paper of Hofstede [45] an assumption was made that the value of «Long Term Orientation» reflects “strategic thinking" specific for given businesses and is denoted as lto and largely determines the Keynesian “propensity to save”. Below we assumed that the propensity to save is proportional to this parameter: $s \approx 0.005\,lto$.

In Fig. 4.5 [52] are compared values of the economic mentality parameters for three developing countries: Ukraine, China and Latvia. The share of investment in fixed assets in 2008 in these countries: Ukraine's GDP - 26%, China - 40%, Latvia - 30-35% [53].

An analysis of scientific publications shows that existing economic models, taking into account psychological characteristics of economic agents, are based primarily on econometric basis. An attempt to apply the ADM as a tool for studying the influence of mental factors on the dynamics of economic growth was made in [54].

In this study the most important parameters that determine the economic mentality were selected, a multi-agent ADM model of economic growth, taking into account these factors was developed and comparative analysis of the impact of different mental characteristics of the agents - the subjects of economic activity in the potential growth rate of the economy and its social and economic aspects provided.

Below we present the main results of this study, related to the impact of economic mentality factors on economic dynamics on the basis of the developed ADM version of the economic growth model [54].

Description of the agent based model. The ADM-model of the economy, adopted in [54] encompasses a number of processes (competing production companies) producing a single aggregated good, and their interacting A-owners and hired A-workers agents. The economy is closed in the sense that the purchasing power of the population is determined by its labor income.

The initial conditions for the capital assets K and for labor force L of a company are $K(0) = K_0$ and $L(0)= L_0$. The output of each company is determined by two-factor Leontieff production function:

$$Y = \min(\frac{K}{a}, \frac{L}{b}) \quad , \qquad (4.1)$$

Here coefficients a and b are reflecting, respectively, the productivity of capital and labor. The following model variables were set: $p = \frac{1}{b}$ – labor productivity; $f = \frac{1}{a}$ – capital productivity; k = K/L - capital-labor ratio; n – population growth rate; μ – capital depreciation rate; s – propensity to save;

Mental charactersitics: lto – for strategic thinking; rat – for rationality; off – for part of the income withdrawn from the economy by the entrepreneur.

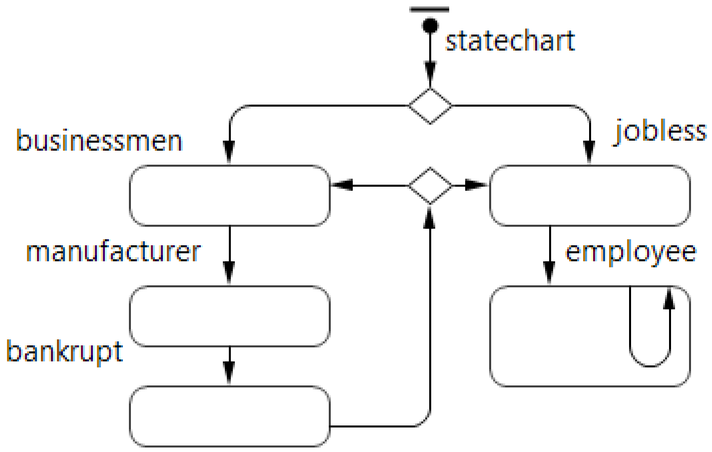


Fig.4.6. Agents statechart

At the beginning of simulation population of agents is divided into entrepreneurs (A-owners) and hired workers (A-workers) in the ratio close to official statistical data [35]. The statechart (diagram of all possible states) of agents is presented in Fig. 4.6.

Possible states of an A-owner are:

- businessmen – self-employed person;
- producer– entrepreneur having working enterprise;
- bankrupt – entrepreneur, whose company became unprofitable;

Possible states of an A-worker are:

- jobless – unemployed;
- employee – employed.

The objectives of the A-owner are:

1. Hiring employees, investing in fixed assets for expansion of his business;

2. Maintaining effective demand (consumption of goods is not guaranteed);

3. Distribution of the incom between productive accumulation, wages, and profit (withdrawn from the business).

It is assumed that each A-owner has a start-up capital, allowing him to launch the company and hire a number of employees. Since capital-to-labor ratio and productivity are exogenous parameters, same for all businesses, the potential rate of development of each company is determined by strategic thinking and rationality of entrepreneur.

The value of *lto* (strategic thinking) parameter is set by a random variable distributed according to the normal (Gauss) law. Dispersion of mental manifestations characterizes the variability of the characteristics of the population. This characteristic is decisive in the choice of an entrepreneur starting value of accumulation rate *s*. Agent-entrepreneur can adjust this value after assessing the current situation on the market.

Rationality of the entrepreneur is represented by Boolean variable rat. The value of *rat* is '*true'* for rational and '*false*' for irrational A-owner. Irrational A-owners are seeking for any possibility to withdraw a part of their income from the economy. To estimate the part of the income withdrawn the NGO Tax Justice Network data have been used[9]. An assumption was made that irrational A-owner withdraws from its business the same part of his income, which is equal to the share in GDP of country's capital runaway to offshore zones.

ADM model, which allows assessing the impact of mental factors on the rate of economic growth, was implemented in AnyLogic software environment [6]. The external economic environment was described in terms of the Solow model [7]. We considered one-sector economy, producing a single aggregated product that can be consumed or invested. The state of the economy at time *t* is determined by the following variables:

Y^t – output (gross domestic product);

K^t – capital;

L^t – labor force;

[9] This official website contains information about the amount of country's capital withdrawn to offshore zones [31].

C^t – non-production consumption fund;

I^t – investments.

The exogenous variables:

ν – labor force growth rate;

μ - rate of depreciation of fixed assets;

s_n – propensity to save;

Note that the rate of of accumulation *s* is not a rigorously exogenous value, its value is affected by the A-owners, depending on the situation on the labor market and on personal preference in the distribution of the company's income. Output is determined by the two-factor Leontieff production function Eq (4.1) with fixed proportions of the factors of labor and capital [31], *i.e.* labor and capital factors are refarded as not interchangeable.

The aim of the simulations performed was the study of behavior of the A-owners performing the distribution of the business output between consumption and investment (the A-owner's decision). It is supposed that rational A-owners will pay such salaries which are needed for households for consumption of produced aggregated goods (households supposed to have no savings, which could be turned into investments), and A-owners invests in the production, bypassing the household.

The period of the model time corresponding to one month of the real time is called 'step'. All A-owners produces the same aggregated product. Produced, but unsold goods are a cost of the producer; there are no investments in stocks. The investment part of the output is produced by A-owner in the amount required for the looked-for expansion of his business.

Formation of market relations is represented by multi-agent ADM of economic system. Two types of agents are considered: A-workers or A-owners. The total number of agents N_Σ is sum of the numbers of A-workers N_W and A-owners N_B. Properties of *i*-th A-worker agent (i=1,…,N_W) are set by the state vector at time *t*:

$$S_i^t(W)=\{A_i,\mathrm{w}_i\},$$

where A_i - economic activity; w_i - salary of A-worker.

Properties *i*-th A-owner agent $(i=1..N_B)$ at time *t* are descibed by state vector:

$$S_i^t(B)=\{A_i,h_i,W_i,V_i,N_i,k_i\},$$

where A_i is his economic activity; h_i is "thrift" of the A-owner; W_i - salary, proposed by the A-owner; V_i - size of the business; N_i - number of A-workers; k_i - capital-to-labor ratio.

At the time of launching of simulation the population of agents is subdivided into A-owner (producers) and hired A-workers based of their economic activity. A-owners independently realize: 1) hiring (firing) A-workers; 2) expansion (or reduction) of the production output; 3) distribution of their income. The behavior of A-owners prioritizes in distribution of income between: 1) investments in production, 2) salaries of A-workers, and 3) personal enrichment. The challenge of A-workers agents is finding the best offer on the labor market.

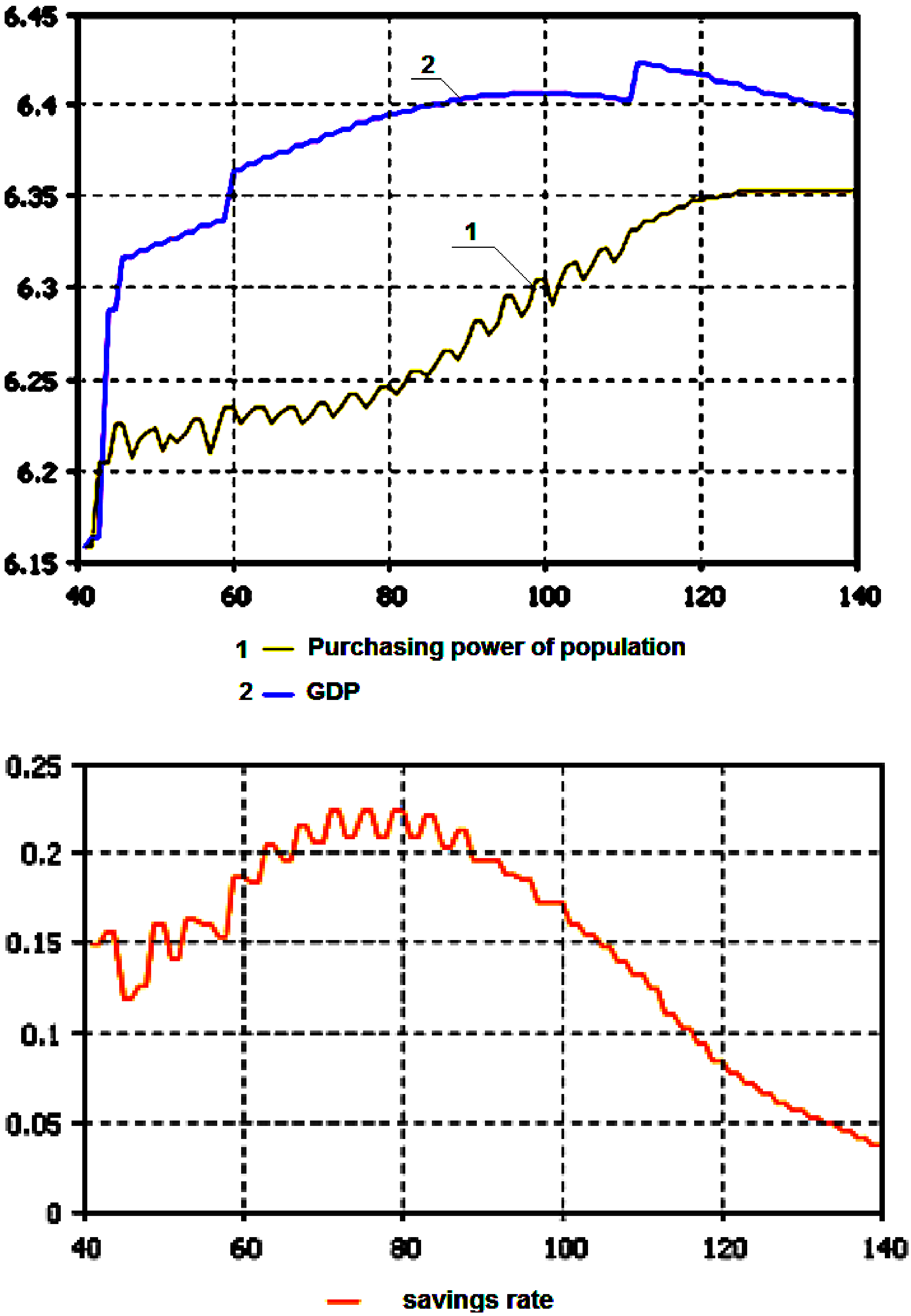
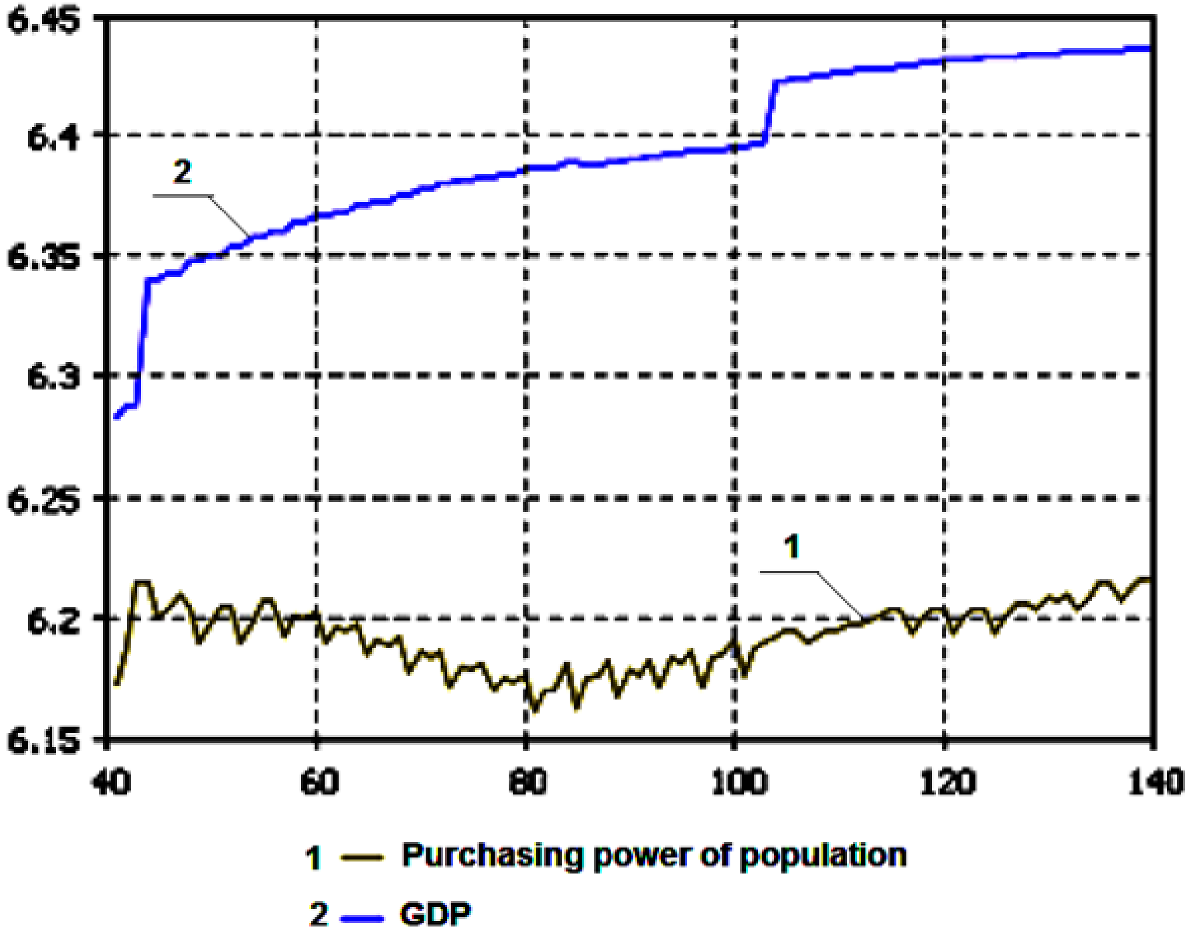


Fig. 4.7. Results of simulation at s=0,1



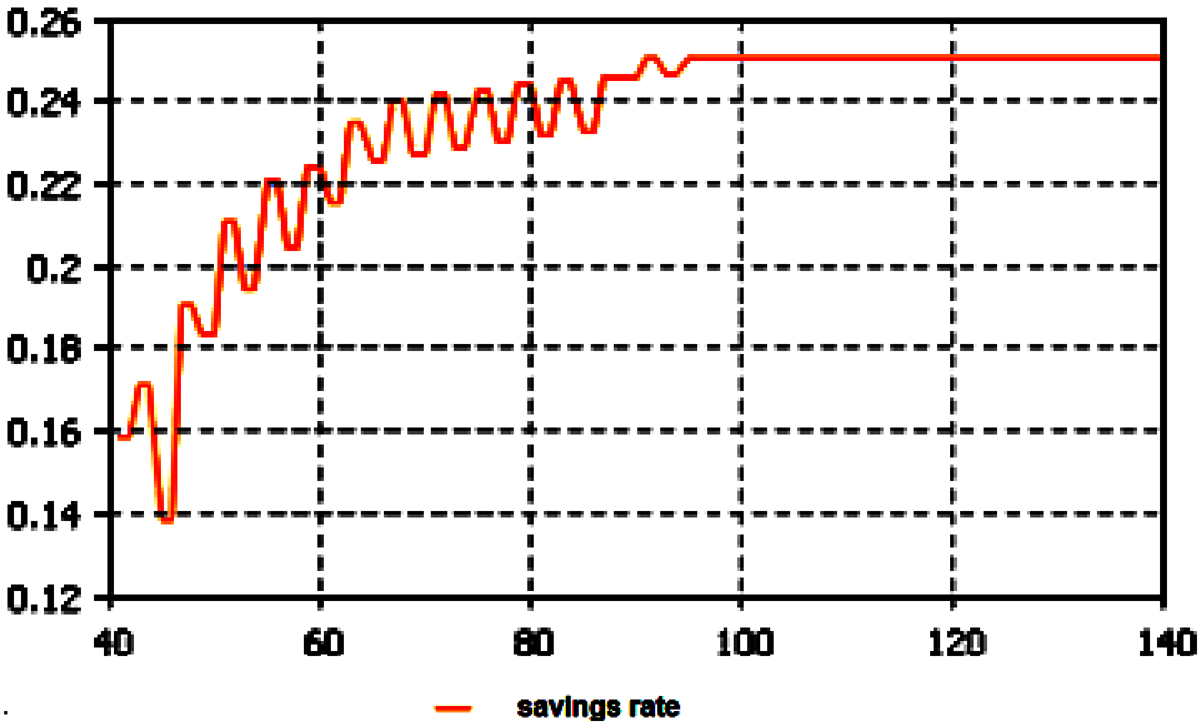


Fig. 4.8. Results of simulation at s=0,25

Income obtained after the realizaqtion of output is distributed by A-owner between the wages, investments and personal enrichment. Since the ADM-economy is closed, the purchasing power of the population determines the effective demand; there is a conflict between the aspiration of A-owner to increase profits and interest in growing of the purchasing power of the population. The process of resolving this conflict is largely determined by the mentality of economic agents.

The model described above has been implemented in AnyLogic environment. A number of computer simulations based on this model have been performed [32].

The results of the first simulation are shown in Fig. 4.7. Salaries are rising at the expense of the savings rate. Rapid growing of wages is, at the same time, the end of economic growth. Low savings rate leads to degeneration of the capital and lower GDP.

Fig.4.8 shows results of another simulation, in which A-owners tend to keep savings rate s at a level of 0,25 instead of increasing salaries. Lowering of wages may lead to a short-term reorientation on the export economy. Growing, due to the high savings rate, industrial capital enables the economic growth and gradual improvement of people's purchasing power.

The aim of the second series of simulations was the study of output dynamics at different values of the rat parameter. Model parameters were evaluated using Ukrainian statistical data [35]. The following values of the model parameters were accepted: $p = 6400$, $f = 0.34$, $k = 18500$, $n = 0.02$, $\mu = 0.03$, $lto = 55$ (average) with dispersion of 0.01, and $off = 0.083$.

The off value was estimated on the basis of statistical data analysis [35] and Tax Justice Network data, according to which during last 23 years 167 billion USD have been transferred from Ukraine to offshores [61]. The value of lto parameter was adopted from the study of Hofstede [52].

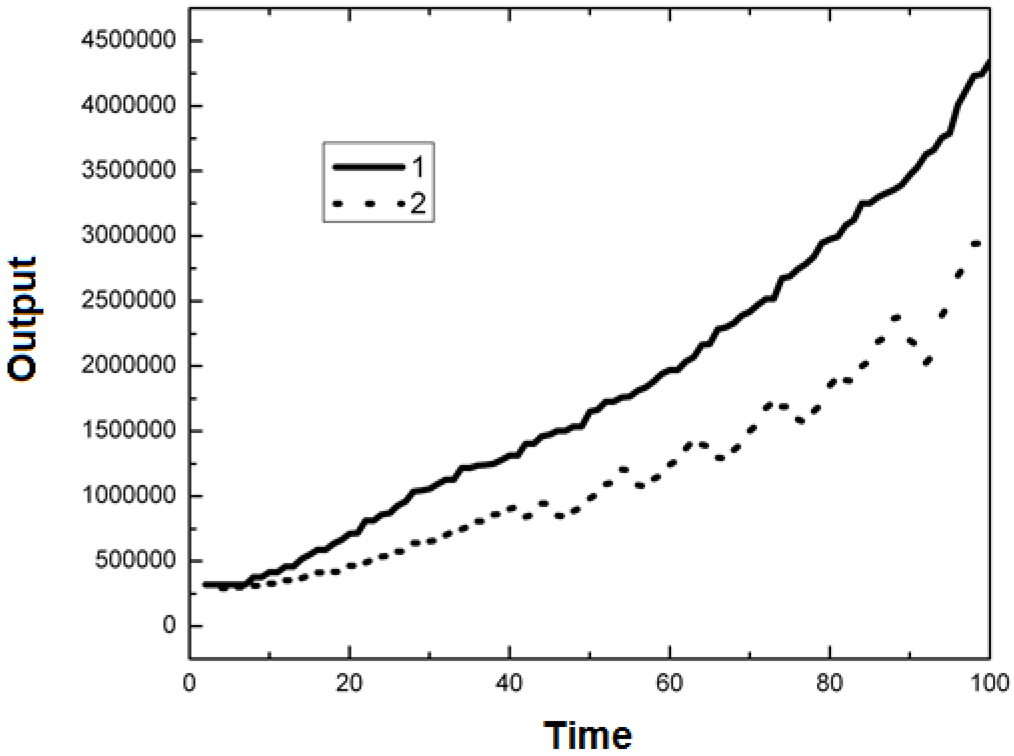


Fig.4.9. Dynamics of production at the rational and irrational behavior of the A-owners.

As can be seen from Fig.4.9, the rational behavior of A-owners (curve 1) provides a stable economic growth. Depressions of the economy, where many irrational businessman are present (curve 2), is none other than a crisis of overproduction caused by the limited purchasing power of the population.

The aim of the second series of four simulations was the identification of the economic system reaction on the behavior of mixed populations of A-owners, having different mentality parameters.

The following parameters were kept unchanged in these simulations:

$p = 6400$, $f = 0.34$, $k = 18500$, $n = 0.02$, $\mu = 0.03$.

Were changed only the parameters that reflect the mentality factors:

1) lto=83, rat=*true* for all A-owners (set of parameters specific to the German entrepreneurs);

2) lto=70, rat=*true* for 75% A-owners; off =0.067;

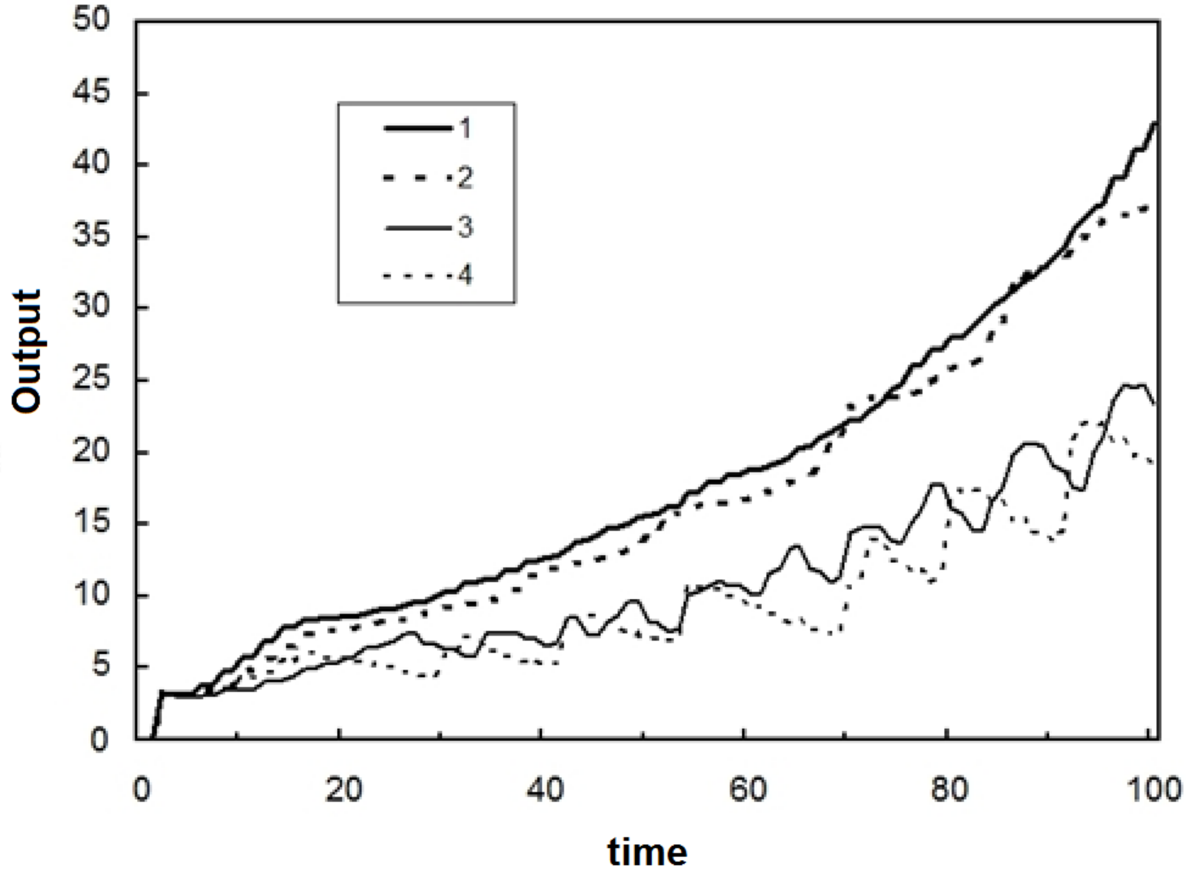


Fig.4.10. Dynamics of production in different communities

3) lto=55, rat=*false* for all A-owners; off =0.083 (set of parameters specific to Ukrainian businessman);

4) lto=70, rat=*false* for all A-owners; off =0.067 [10].

Economy, the rate of growth of which is displayed by curve 3 in Fig.4.10, significantly gives in to the economy in which agents-entrepreneurs have a set of parameters specific to the German entrepreneur (curve 1) due to the irrational behavior of A-owners and smaller values of lto parameter.

Curve 4 shows that the high propensity to save provides a higher rate of growth only when combined with rational behavior of A-owners. Irrational desire of the A-owner to maintain the level of wages in the range close to the subsistence level, leads to crisis (curves 3 and 4).

[10] Parameters in the second and fourth simulations are set conditionally.

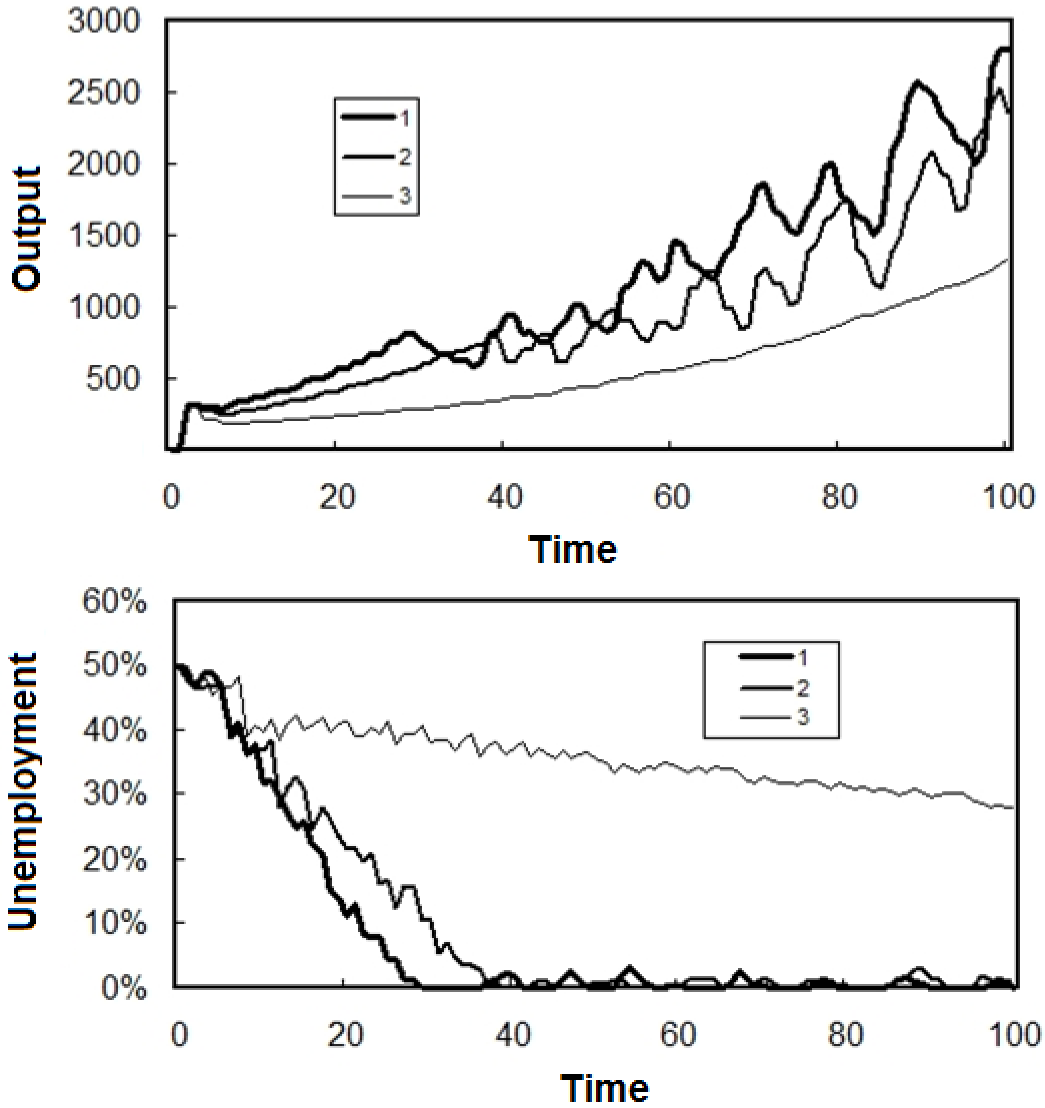


Fig. 4.11. Influence of the runaway capital to the dynamics of output and unemployment

Analyzing the behavior of the curve 2 it may be concluded that only a large proportion of rational A-owners is capable of holding the economy from the crisis.

In the third series of simulations there was investigated the dynamics of output and the unemployment rate according to the proportion of capital output. Unchanged were parameters: $p = 6400$, $f = 0.34$, $k = 18500$, $n = 0.02$, $\mu = 0.03$, $lto = 55$, $rat = false$ for all A-owners. The only parameter *off* was varyed: 1) $off = 0.083$; 2) $off = 0.14$; 3) $off = 0.33$.

The curves of the 1st and 2nd simulations (Fig.4.11) indicate reducing the rate of economic growth and reducing unemployment by increasing the share of runaway capital. Results of the 3rd simulation demonstrate the absence of the crisis in the economy: the output is growing smoothly, overproduction, as a result of limited demand on the part of the employed population, is absent. At the same time the high unemployment, ensured by such a "fair distribution of the income", can be a catalyst for social unrest.

4.3. ADM of socio-economic relations at the enterprise

Another example of the use of agent-based approach to the study of social and economic phenomena is an ADM model of emergence and resolution of the labor conflicts at the company, developed in AnyLogic [54]. The development of the conflict depends on the mental characteristics of participants and is initiated by a low wages (in the workers' opinion). The strike begins initiated by the trade union in the case when the number of "dissatisfied" workers (determined by their mentality factors) exceeds a certain limit. Strikers must have a strike fund enough to survive the strike period which will not be paid over by the administration. The size of the strike fund is the main factor that determines the duration of the strike. The A-owner of the company also can respond differently to the demands of the strikers. First, they must estimate the share of wages in the cost of production. If this "share" is large enough, and the A-owner does not consider possiblity to increase wages, he sends 'parliamentarians' to the strikers, or waits until the strikers will exost the strike fund. In the case when "wages share" is small enough, the A-owner can accept the demands of the strikers (of find a compromise: a partial satisfaction of strikers demand).

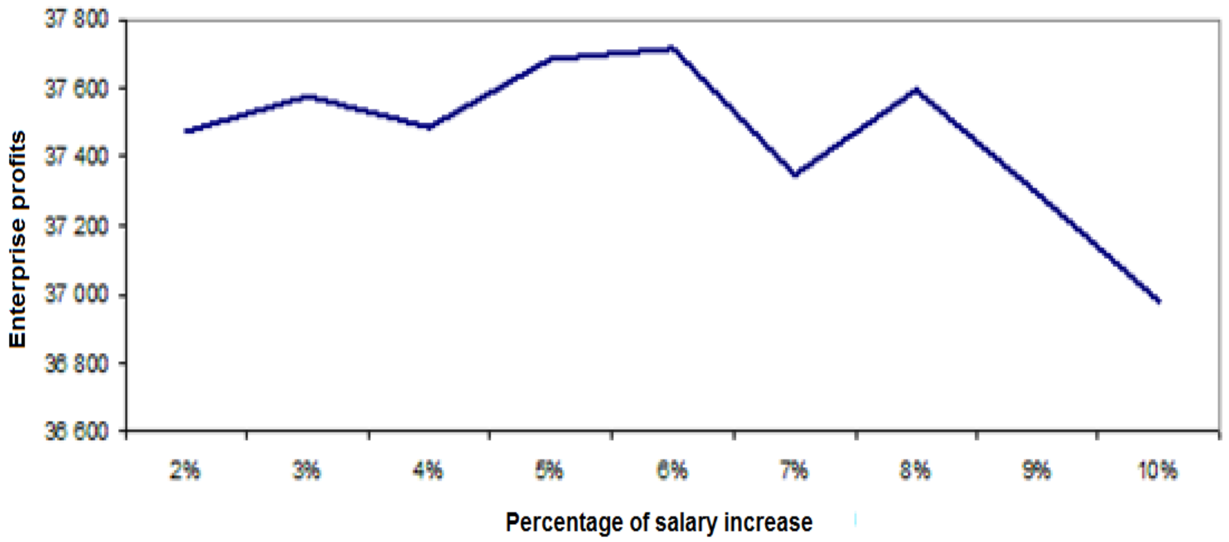


Fig.4.12. Company profits at various strategies in solving conflicts of the A-owner.

The trade union may also accept or reject proposals of the A-owner to finish the strike. Terms of the agreement are determined by parameters, which reflect the mental characteristics of society.

The state of the A-worker can be described via statecharts used within AnyLogic. In this case, an A-worker can exist in three states: 1) satisfied, when wages correspond to its needs; 2) dissatisfied with wages; 3) on strike.

Transitions from state to state correspond to the following possible steps:

1 - A-worker is not satisfied by his wages;

2 - A-owner of the company increases wages, avoiding a strike;

3 - A-worker decides to participate in the strike;

4 - business A-owner meets demands for higher wages as a result of the strike;

5 - strike stops, but the workers are not fulfilled.

<u>Parameters of the model and their notaitions:</u>

• Employees number - N;

• The current salary of A-workers (generated for each agent a random variable uniformly distributed in the range S_0 - S_1);

• Monthly payments (% of salary) in the strike fund - D;

• Necessary (in A-worker opinion) salary range S_1 - S_2;

• Necessary conditions for the strike: a) the portion of dissatisfied by their salary of the total number of workers exeeds a specified level M; b) the size of the strike fund is at least F;

• Duration of strike (days) - T;

• Minimum interval between strikes (days) - I;

• Quarterly profit - P;

• Losses due to one day of strike - L;

• The percentage of the salary increase as a result of a successful strike - X.

There have been performed a number of sinulations, with different per cents of the salary increase and the loss incurred by the company due to its inactivity were estimated.

The corresponding values of model parameters were: N = 100, S_0 = 2000, S_1 = 3000, S_2 = 4000, D = 1%, M = 30%, F = 15000, T = 5, I = 90, P = 4500000, L = 50000, X varyied from 2% to 10%. The results of simulation are illustrated in Fig.4.12.

As seen in the chart (Fig.4.12), with a small percentage of salary increase the profit decreased due to frequent strikes. Then there is a series of X values at which the profit value reaches maximum. Further concessions by the A-owner are likely to reduce profits, while increasing of payroll reduces the income more than just stopping of the production process.

The results indicate the efficiency of the proposed model. The prospect of further improvement of the ADM model can be achieved by addition of elements of game theory to the behavior of agents [54].

4.4. Modeling of economic dynamics at conditions of labor shortage

An another example of the ADM multi-agent approach, implemented within the AnyLogic environment, is the investigation of economy at conditions of limited HR, which can serve as an additional example of a simple model reflecting the role of the mentality in the economic development.

The key point of this model is the subdivision of all agents on potential business A-owners and wage earners. It is known that only a small part of people have mental qualities (activity, mobility, and risk tolerance), necessary for business. Just this “share of business-active people” appear here as the model parameter reflecting mental characteristics of society.

Qualitative results of computer simulation, showing the dynamics of output as a function of this part of active agents are shown in Fig.4.13. If this share in the society is low, the development of economy is smooth and the output (curve 1) reaches a maximum at full employment.

A-owner takes occasionally loans, which are necessary to expand his production facilities and the jagged dependence of the output reflects the periodicity of this process.

When the share of active agents (potential entrepreneurs) in the society grows, their struggle for limited labor resources is becoming a significant factor. The A-owner, who has decided to develop his business under conditions where there is no free HR, is forced to "poach" A-workers from his competitors. This requires increasing wages and the size of the the wage fund. Such businessman risks not returning the bank credit at time, and goes bankrupt. This perspective determines the actions of this agent-entrepreneur and his decisions.

Agent-worker, in turn, may react differently to the proposal to change his work place. The probability of acceptance of such proposition is also determined by his mental characteristic (mobility).

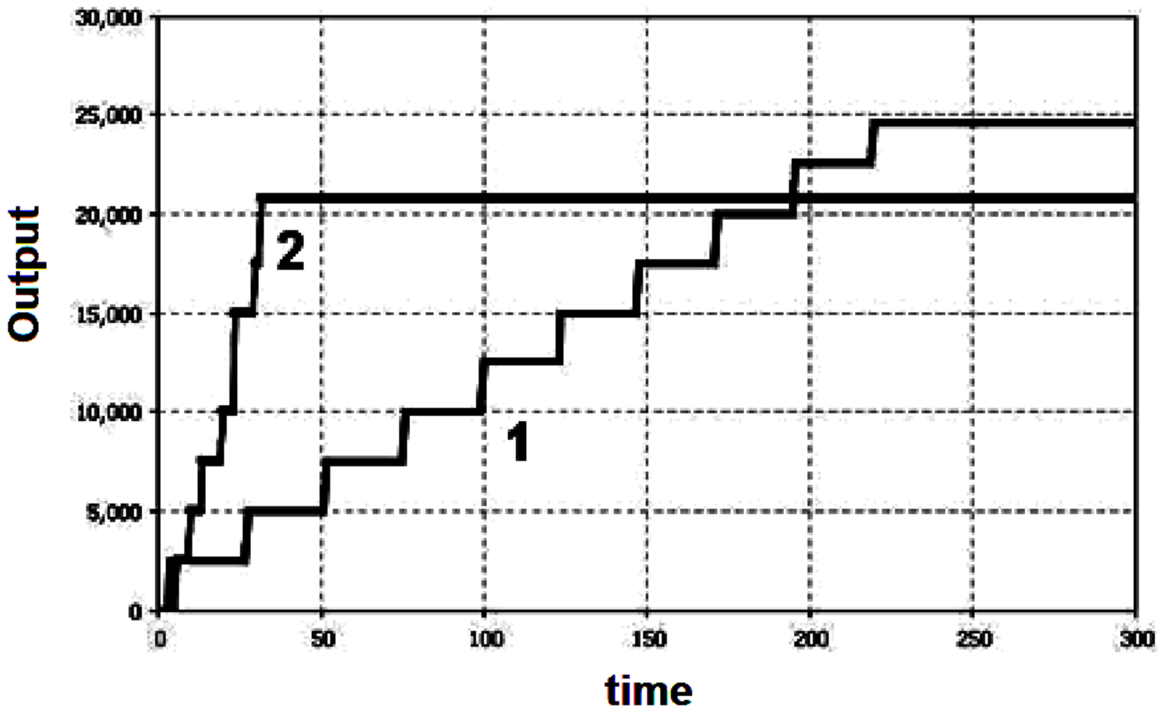


Fig.4.13. The dynamics of economic development in conditions of labor shortage

Economic development in a society with a greater proportion of potential A-owners (with a more developed small businesses) is more intensive, but the increase in the number of A-owners reduces the number of workers (at a constant population), and the output does not reach the maximum possible in the previous case (curve 2 in Fig.4.13) [67,68].

The next step in the development of the model is to bring conditionally taken parameters to the sociologically proven data. This will simulate the real demographic situation in Ukraine and the dynamics of the current pension system, to specify the level of wages on the basis of Ukrainian statistical data and to take into account the "semi-legal" sector in which the "in-envelope" payments are an addition to the official salary.

On the basis of such a model it is possible to solve a number of practical problems, including studies of the role of reforming the tax and pension systems and the conditions for maintaining the stability of the socio-economic relations in society.

4.5. Modeling of the Pension Fund dynamics

One of the most acute socio-economic problems in Ukraine in recent years becomes the need to reform the pension system. This problem concerns almost every citizen of the country. The existing solidary pension system, based on the principle of solidarity between generations (when current A-workers are actually paying pensions to today's pensioners) leads to the continuously growing deficit of the Pension Fund. The increase in life expectancy and reduced fertility has led to aging of the population, which in turn has caused disbalance of the solidary pension system. Working payers of pension contributions (social tax) can no longer keep the growing number of pensioners. When current trends continue the solidary pension system expects financial instability. This prospect has caused the need for pension reform in Ukraine.

In [66] an agent-based model has been developed and applied to study the stability of the solidary pension system. It takes into account the impact of the employee mentality at conditions of demographic crisis and significant portion of informal sector of the economy. Application of ADM approach for simulation of Pension Funding allows to create an artificial community, imitating the behavior of a real community of the working population of Ukraine. It allows to identify the most significant factors (demographic, socio-economic, *etc.*), affecting the stability of the real the pension system of Ukraine.

Among many factors influencing the solidary pension system, the most important are: demographic crisis and presence of a large informal (shadow) sector in Ukrainian economy.

In 1993, the maximum number of 52.2 million people was registered in Ukraine. In subsequent years there has been a steady decline in population.

According to the State Statistics Committee on November 1, 2009 the population of Ukraine was 46 million people [35]. Study carried out by World Bank in 2009, predicts further decline of the Ukraine's population in 2025 by 24%, or by 12 million people [56].

Twenty years have passed since the collapse of the Soviet Union planned economy. They are characterized by essential changes in forms of employment in favor of increasing the proportion of its informal component. This reduced the amount of mandatory pension contributions A-owners of enterprises and A-workers. An imperfect and corrupt tax system makes the A-owners of small and medium-sized businesses looking for ways to tax evasion. Mandatory taxes and charges on the wage fund are traditionally “reduced” by formal decrease of the worker salaries to the minimum wage or by using an inofficial employement. In the first case, additional inofficial paymentsare added to the minimal official salary, and in the second case - the entire salary to be issued "in the envelope".

ADM of socio-economic relations in the company, accounting the demographic dynamics in Ukraine and the mentioned above mechanisms of the employment choice, was implemented in the software environment AnyLogic [66].

This model considers interaction of two classes of agents: agent-government and agents-employees. The functions of the government-agent are:

1) Collection of taxes (pension contributions)

2) Pension provision,

3) Inspection of tax payments,

4) Gathering of statistical information.

The statechart diagram of A-worker describes his behavior over the whole life as a sequence of possible transitions from one state to another. The "life" of an agent is modeled by transitions between the following states:

1) Childhood;

2) Unemployed;

3) Work in the formal sector;

4) Work in the informal sector;

5) Continuation of work in the informal sector after a fine;

6) Staying retired.

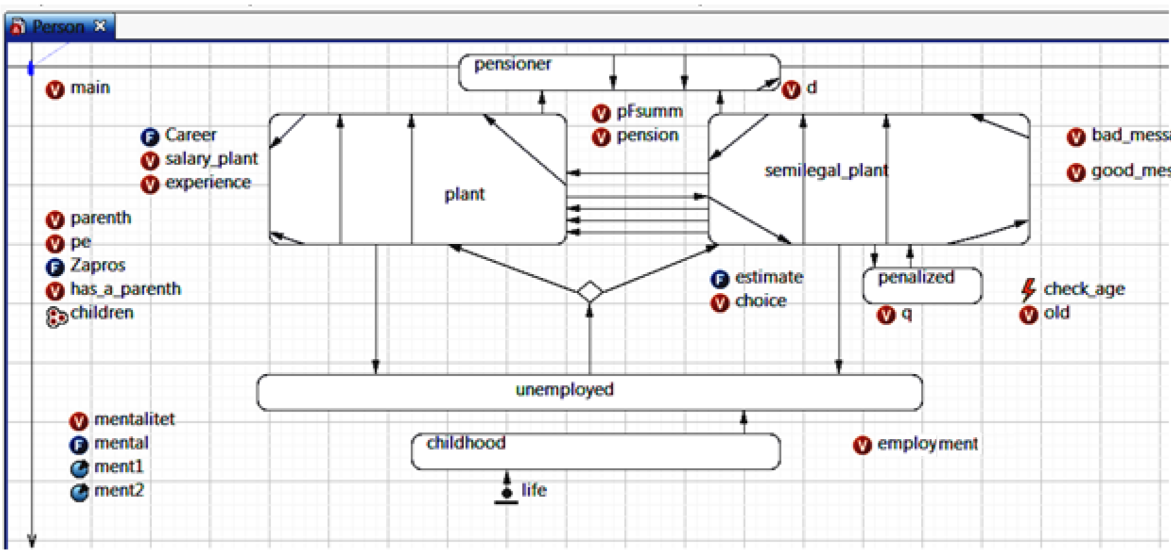


Fig.4.14. The statechart diagram of an A-worker.

The life cycle of an A-worker starts with the birth and childhood. At this stage, he inherits the mentality of his parents. After a certain period of being unemployed the agent gets an employement. He can work in both formal and informal sectors. From the A-worker point of view at each option has its own advantages and disadvantages.

First type company (with formal employment) offers: 1) social package, 2) career prospects, 3) additional payments for years of service, 4) a pension after retirement. Joining the second type company (offering inofficial employment) gives to employee significantly higher salary, but also 1) possibility of fining by the government, and 2) higher probability of arbitrary dismissal. The second type company provides no pension contributions for the employee, and upon reaching the retirement age such a worker remains on his own, without pension.

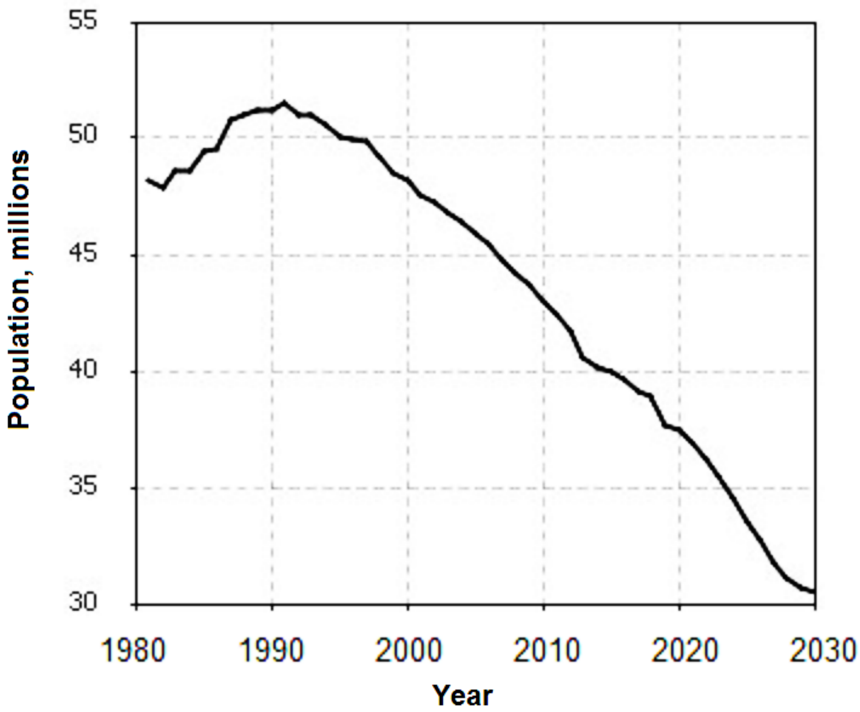


Fig.4.15. The dynamics of population in Ukraine (predicted).

Choosing of the company type (legal or shadow) is largely determined by the employee mentality. The model takes into account: 1) necessity of pension contributions payment by agent from his official salary received in the legal company, 2) salary in the "envelope" (without pension contributions) and 3) a combination of both. A definite probability of the employee dismissal exist, followed by repetition of the employment phase is set in the model. Reaching the reproductive age, an agent gives birth to posterity. Upon reaching the retirement age this agent retires. Some of the retired agents are able to find additional earnings in the informal sector.

A-workers in their mentality roughly divided into three types, each of which in their own way relates to the ability informal employment.

The definition of informal employment we use below is largely based on the definition of the informal economy, proposed by E. Feige [61]: "The informal economy covers the economic activities that are associated with the departure of the costs and benefits and are excluded from the rights contained in the laws and regulations of legal the documents governing the relations of property, licensing, labor relations, torts, financial loans and social security system". Thus, the scope of the informal sector does not cover criminal activities related to illegal production of goods or services.

Agents of the first type are the part the working population, which believes that big earnings here and now is much more important than a decent pension. Their choice in their own way is reasonable. Mistrust of the government caused by an instant impoverishment of the population after the collapse of the Soviet Union, forced a significant part of disaffected people to rely on own efforts. High salary is the sole criterion by which this group of agents are choosing their job. Upon reaching the retirement age or on premature retirement such a worker can not rely on help from the government.

Agents of the second type are able to learn and draw conclusions based on their own experience. The desire for higher wages often leads them to the informal employment. However, fines, dismissal and fear of being left without a pension, can encourage them to move into the formal sector. The presence in the family of at least one member who is in the formal employment sector, and already has the appropriate privileges in the form of pensions, insurance and benefits, is a motivation to move into the formal employment for other family members.

Agents of the third type are traditionally favored official employment in the formal sector. They are part of the population always employed on public enterprises and institutions. Their priorities are social protection, higher pension, various benefits and allowances.

In the model, the following assumptions are made:

1) the size of wages in the informal sector is in average by 40% higher than the wages received by workers in the formal sector;

2) career, allowance for years of service are possible only in the formal sector;

3) there is no increase in population due to migration.

In Figs.2* and 3* are presented the results of computer simulations with a mixed population of agents, when the portion of each type is equal to one third of the total number of agents.

Despite some arbitrariness of such situation, the demographic situation of the last decades in Ukraine can be clearly seen in In Figs.2* and 3*. The maximum in population observed approximately in 1993 was the last echo of the baby boom of the early 80s, after which the population began to fall.

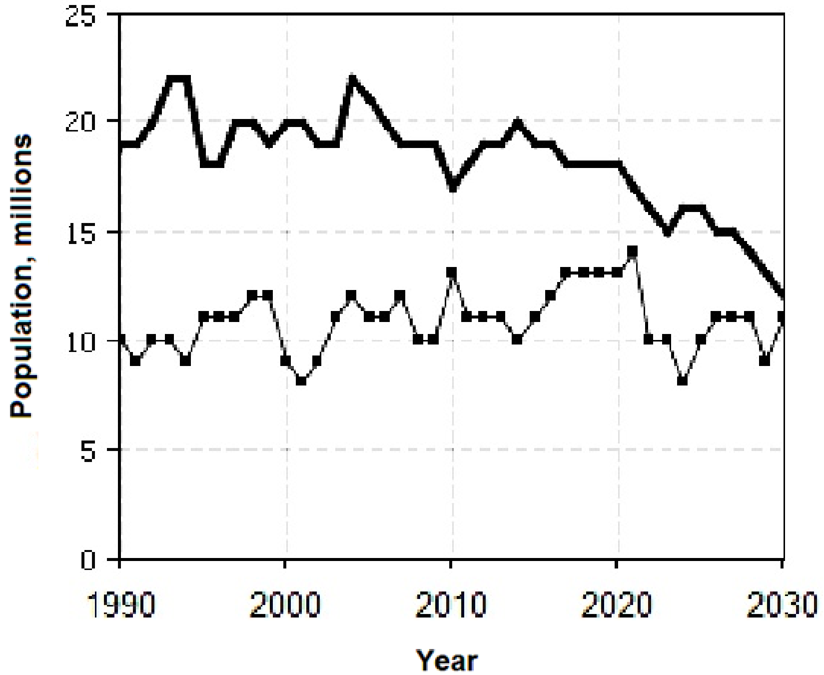


Fig.4.16. Dynamics of the number of pensioners and taxpayers.

All forecasts of the negative trends according to study performed by the World Bank, are fully confirmed in the simulation [56]. The curves in Fig.4.16 reflect the dynamics of the actual number of pensioners and payers of social contributions. Here only workers in the formal sector are taken into account. There is a tendency to decreasing of the ratio of workers to pensioners.

An essential improvement of the financial condition of the Pension Fund can be achieved by withdrawing of the informal sector from the "shadow", as well as by stabilization of the population fertility.

At this stage, the developed model has passed the validation stage. The illustrations show a a correct reflection of the demographic situation in Ukraine and the correct reflection of the proportion of the formal sector and show the high quality of the proposed model The next step in application of this model was the stability study of the solidary pension system in Ukraine.

Traditional tax and pension systems in society at conditions of transitional economy may become factors of its social instability. Existing approaches to reforming the social and economic relations of the A-owners and workers on enterprises generally do not take into account changes in the mentality of the population due to processes of transformation.

As was already stated above, study conducted by the World Bank in 2009, predicts that by 2025, the population of Ukraine will be reduced by 24% or by 12 million. This is the worst performance among the countries of Central and Eastern Europe and the former Soviet Union. Such significant decrease in the number of our country's population is associated with both natural factors and with emigration of significant amount of the most active and educated part of Ukrainian society to the more prosperous countries [56].

According to the World Bank study, the portion of people aged over 65 in 2025 will be about 20% of the Ukrainian population (in 2000, this category amounted 14%). This is slightly less than predicted, for example, in Slovenia

(23%), Croatia (22%), the Czech Republic (22%), Bulgaria (21%), Hungary (21%) and Poland (21%). But there is one significant difference: in these countries a market of private pension coverage has already formed, when in Ukraine ihas started, and very slowly, only the first stage of its formation [56]. The above statistics is an important argument for reforming the pension system.

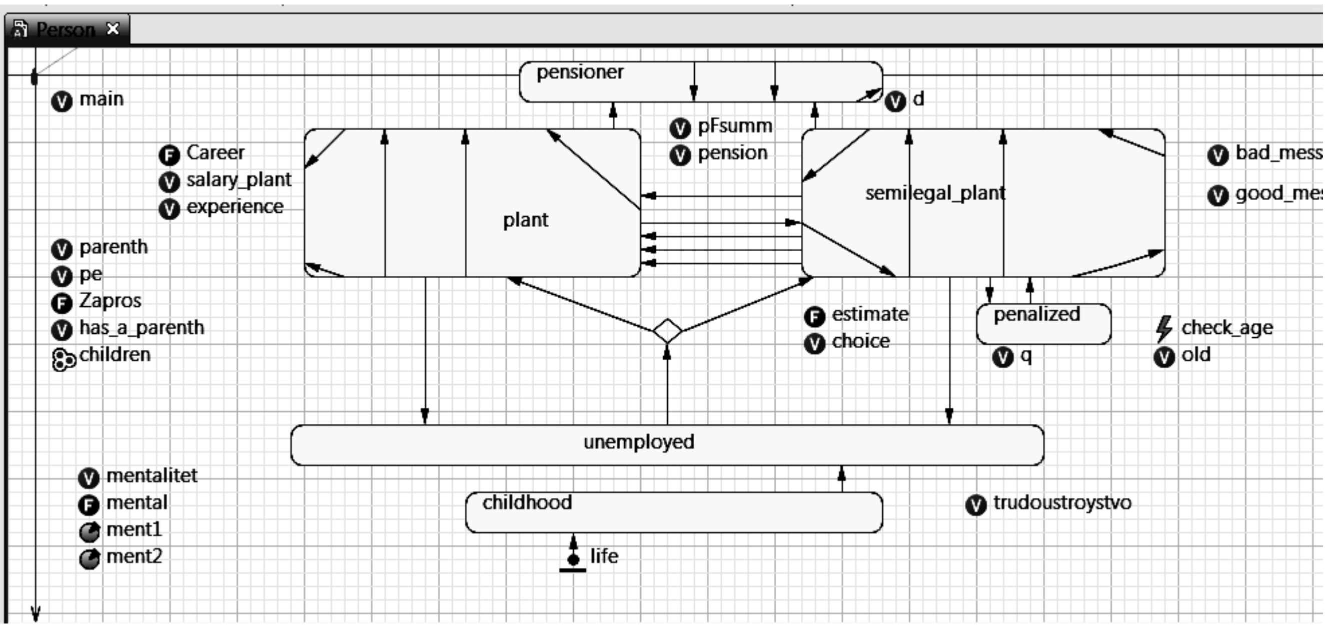


Fig.4.17. Statechart “life of the A-worker”.

To maintain the financial stability, the solidary pension system, functioning in Ukraine, will be forced to reduce public pension benefits, and to increase the retirement age and pension payments.

Solidary pension system is financed by contributions of the working population. One of the causes of untimely and unfair appendage of the pension and social insurance funds is the existence of a vast shadow economy sector, as well as existence of " salaries in envelopes " mentioned above.

Exorbitant tax burden makes the A-owners of small and medium business by any means reduce their tax liabilities to increase (and in the most extreme case, at least not lose) their own profit. Today, employers consider contributions to the pension fund (33.2% of payroll) as too high, especially when compared with other countries, where such royalties, for example, are: 16% in Poland, 19.5% - in

Czech Republic, or 8.75% - in Croatia. They argue that this is a major obstacle to the increase in wages and decrease of the shadow sector of economy [57].

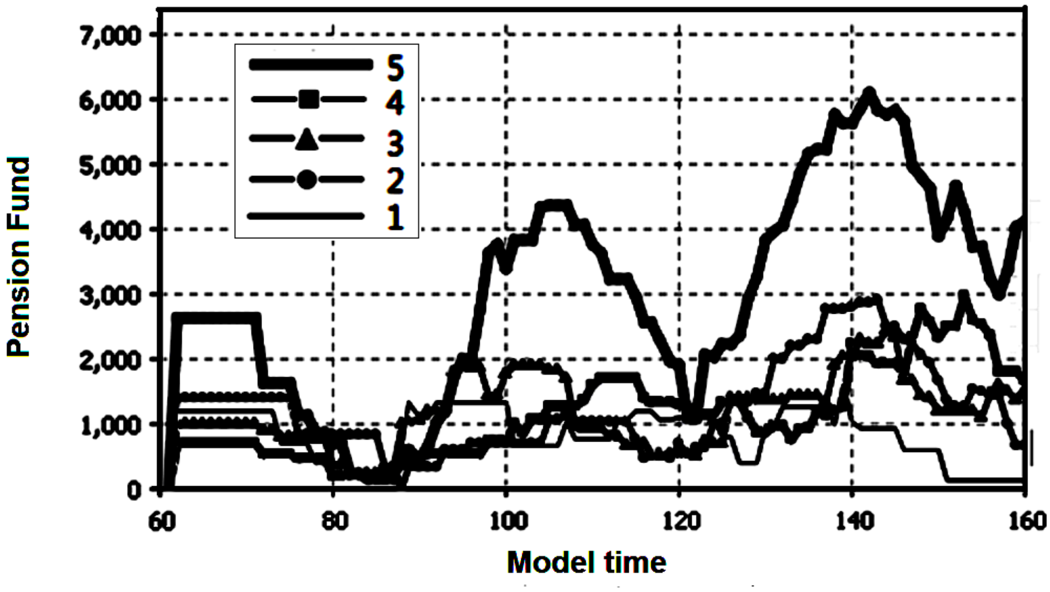


Fig.4.18. Dynamics of the pension fund in a variety of agent-based communities.

The usual way of reducing the tax liability is not register officially the employees at enterprise. In this case, the A-owner's benefit is obvious; his costs are reduced by the amount of compulsory charges on the salary and on the unified social contribution.

Fig. 4.18 presents result of agent-based simulation of the Pension Fund dynamics for a number of communities having different mentality. The size of the Pension Fund (in arbitrary units) plotted here versus model time (in years)[11].

In the first community agents are oriented exclusively on momentary earnings (curve 1). Agents of this community are unable to draw the right conclusions. Their only priority is the maximum income here and now. As expected, the

[11] Here the age of the first agent's retirement is 60.

pension fund was practically destroyed. Only a weak replenishment of the Pension Fund was noticeable due to those agents who have chosen a formal employment with high salary.

Computer simulations with inclined to self-learning agents show their ability to stabilize the Pension Fund. Mistakes committed by agents-workers (or by their neighbours) make them to change their work place, passing from shadow to legal sector, which significant increase in the Pension Fund.

Curve 2 displays the dynamics of the fund in the case when agents do not react to a threat to lose their pension, and do not care on acquisition even a minimum service period needed to recieve pension. A low, in the agent opinion, salary, is forcing them to seek a work predominantly with the higher salary, mainly in the shadow sector. However, the significant amounts of fines or a complete loss of income as a result of the dismissal, may force them to find a legal job with a bit lower salary. Some agents are employed officially, focusing only on the salary, so the filling of the Pension Fund become more intense.

Curve 3 illustrates the case where agents come "out of the shadows," not only because of penalties, but also for the acquisition of a minimum service period for pension. Curve 4 corresponds to the case where the fines, the risk to remain without a stable income in their old age, the desire to gain the necessary seniority needed for the pension - all together makes the agents to move in the legal sector.

When the agents obtain an exceptional confidence in the solidary pension system, the dynamics of the fund becomes positive (curve 5). Agents of this community accept only the regulated relations in the enterprise. Even the offer of a higher salary can not force them to recieve their salary "in envelopes"

Periodicity in graphs is explained by the initial conditions of the simulation. At the time when the simulation starts, all agents are still children. Then the population grows. Retirement of the first agents, of their descendants and of next generations appears after certain time intervals. After the distribution of the agents ages alignes, the Pension Fund dynamics became smoother.

Note that this model contains a number of conventions and assumptions, namely:

- a favorable demographics (1-2 children in the family),

- the life expectancy of 70-90 years,

- a constant level of wages and of the retirement age during simulation,

- no career opportunities in the informal sector,

- wages "in an envelope" are higher the official salaries by the amount of the unified social fee, (*i.e*, on 40% in average).

One of the assumptions is the immutability of the inherited mentality agent, ie, inherited employee personal characteristics, such as reaction on the penalty does not change during the agent's life.

The ADM approach allows creation of communities with different percentages of agents endowed with a different mentality, to vary flexibly the salary, life expectancy, the number of children in the family, retirement age and other parameters. During the simulation, agents respond to changes in the economic environment. For example, changing the ratio of the size of the "black" and "white" wages, leads to a change in the percentage of agents employed in the informal and formal sector.

4.6. Agent-based model of the informal employment dynamics

Presence of a substantial informal sector under conditions of transformation economy is usually caused by weakness of social and economic institutions. Growth of the informal sector is facilitated by several factors:

- reduction of production in key sectors of economics; a low share of wages in GDP;
- low level of social consciousness of the working people;

- high level of taxation of legal business (including high contributions to the government social funds);
- traditional tolerance of society to non-compliance with established laws (anomia [21]).

As a consequence, all these factors lead to the total loss of public confidence in the law enforcement and judicial system of the State [60].

These circumstances are greatly responsible for incomplete formation of the civil society in Ukraine, because mental characteristics of the majority of the population, were formed during the Soviet and even pre-Soviet periods of Ukrainian history.

However, appearance on the forefront of the new generation which grew up in completely different circumstances, can lead to significant and unpredictable social consequences. This is indicated *e.g.* by the bitter experience of social unrest that swept recently throughout the Arab world.

Another interesting field for application of ADM multi-agent approach is simulation of social and economic aspects of in transition economies. Developed on the ADM-basis multi-agent model [66] has been implemented for this purpose within AnyLogic environment and applied to study of the labor resources distribution dynamics between the formal and informal sectors of the economy. Simulated was the same interaction of the agent-government with a variety of agents - employees (private entrepreneurs) as above in Sec.4.5, but the main focus was made on possible changes in the mentality of workers.

The model is based on the following simplifications and assumptions:

1) It considers only two limiting cases of employment: formal employment in the formal sector and informal employment in the informal economy. Informal employment in the formal sector, where exists the "in-envelope" supplement to the official salary was not included.

2) Wages in the informal sector are considered to be in average 40% higher than in the formal sector, *i.e.* on the amount of obligatory social duties on the salary of the employee, as in Sec.4.5. This circumstance forces the agents of the first and second types to move to the shadow sector. Wages in informal and formal sectors are flexible and can vary during the simulation.

3) A-workers in the formal sector receive a bonus for years of service. Agent of the this type, hired on a small salary at the formal sector, after some time, due to allowances for years of service, earns much more, than in "shadow businesses" of the first type, where his career perspectives are completely frozen.

4) The model does not take into account the presence of agents' savings, as well as of income from the secondary employment, bank deposits and other additional revenues.

5) A-workers also give birth to posterity who inherits the mental type of parents.

The ratio of agents with a different mentality varied during the computer simulation. When the simulation starts, agents are distributed randomly, depending on the age and mentality according to the «life» statechart (Fig.4.17). Each agent runs his individual way of life determined by his mentality. At the age of 20 agents become A-workers.

It was assumed that the reproductive age of agents is from 20 to 50, while the number of children in the family can be one or two. The periodicity of the offspring determined by uniformly distributed random variable.

Simulations were carried out with different populations of agents and different ways of the government influence on informal sector. Results of simulation performed using homogeneous population of the first-type agents are presented in Fig. 4.18. At the end of simulation about 63% of A-workers have chosen informal sector as a place of their employment.

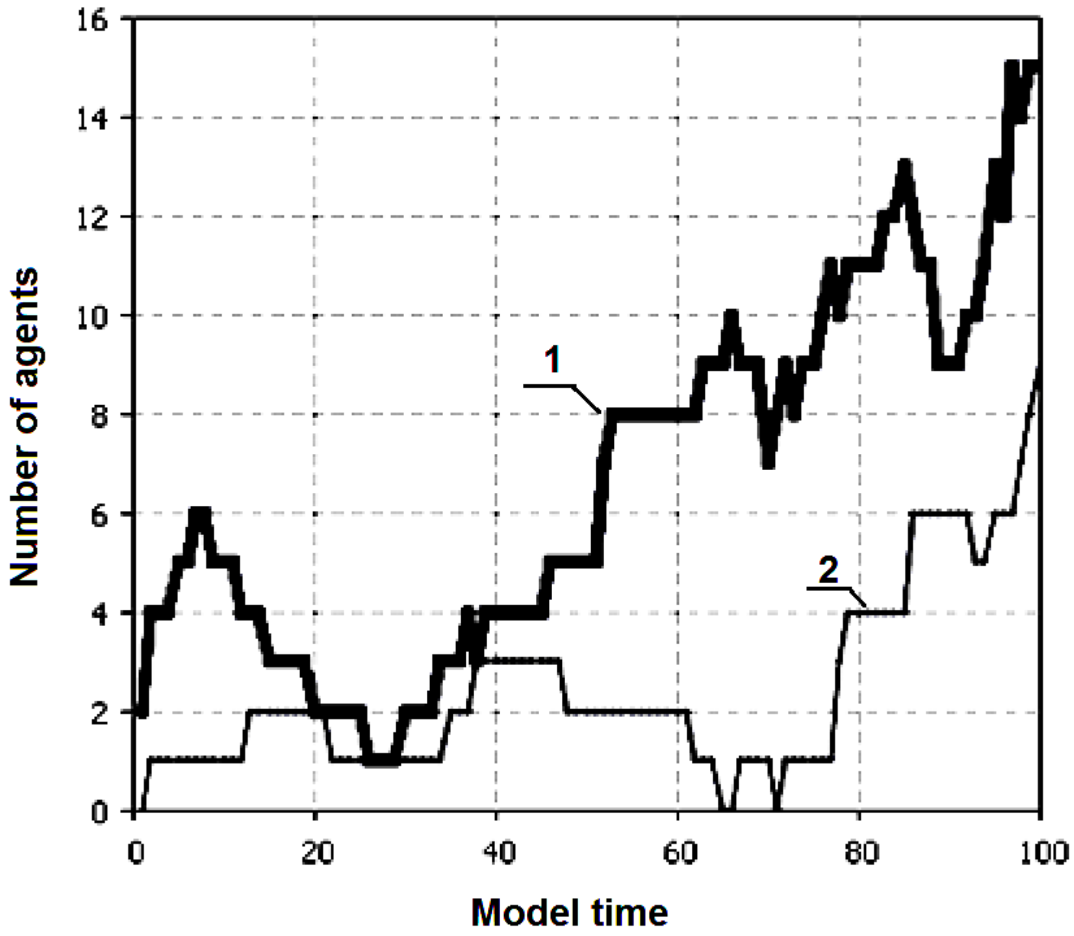


Fig.4.19. Dynamics of employment of the first-type agents (1- employment in the shadow sector, 2- employment in the legal sector)

In simulation with a homogeneous population of the second type agents, upon reaching 45 all of them pass to the formal sector to gain a minimum service period needed for their future pension.

In addition, agents were informed about the size of pension obtaind by their parents (if any), which also led them to avoid the informal employment. The portion of agents employed in the informal sector, at the end of simulation reached only 35%.

In the next simulation effect of government on A-agents was added.

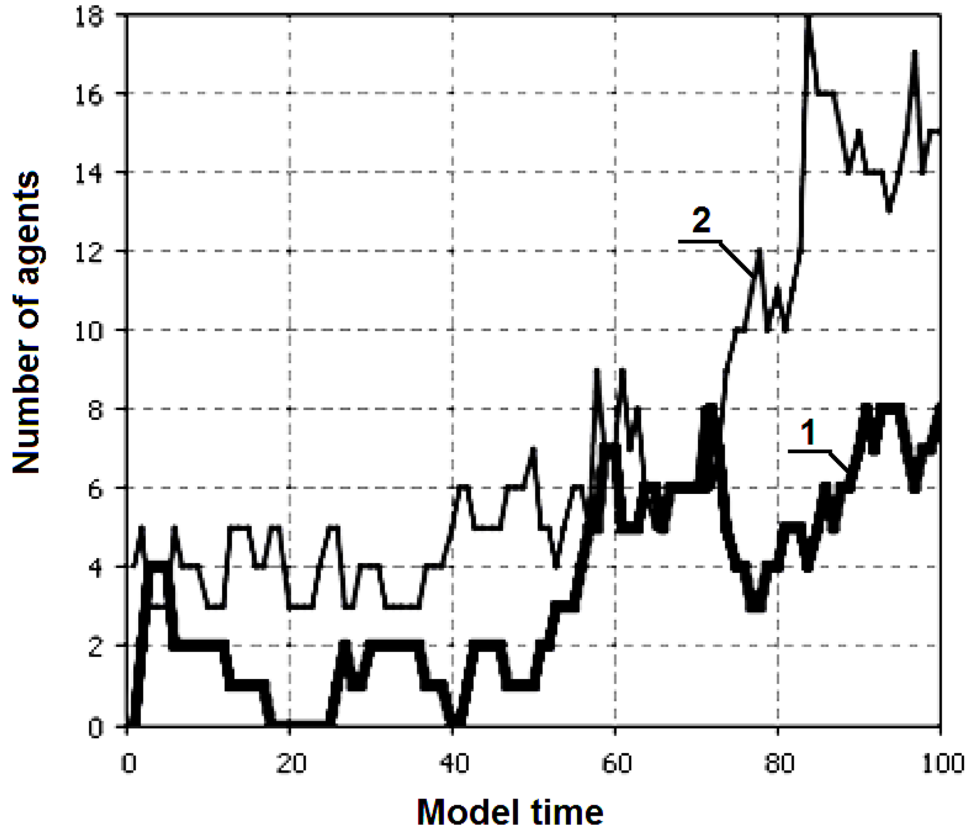


Fig.4.20. Dynamics of employment of the second-type of agents in the case of absence of the government control over the informal sector (1- employment in the shadow sector, 2- employment in the legal sector)

In the case the informal activity is revealed, A-workers are fined by government. The intensity of the government's inspections depends on the portion of informal sector in economy. If this portion is less then 30%, such inspections were carried out every five years, if it exceeds 30%, than inspections are performed every year. In this case A-worker, who was fined twice, is transferred to the formal sector compulsory. Such penalties reduced the size of informal employment to 29% (Fig.4.21).

These results confirm the sensitivity of the model to the composition of the agent-based community (mentality of agents) and adequate response of agents to changing external conditions.

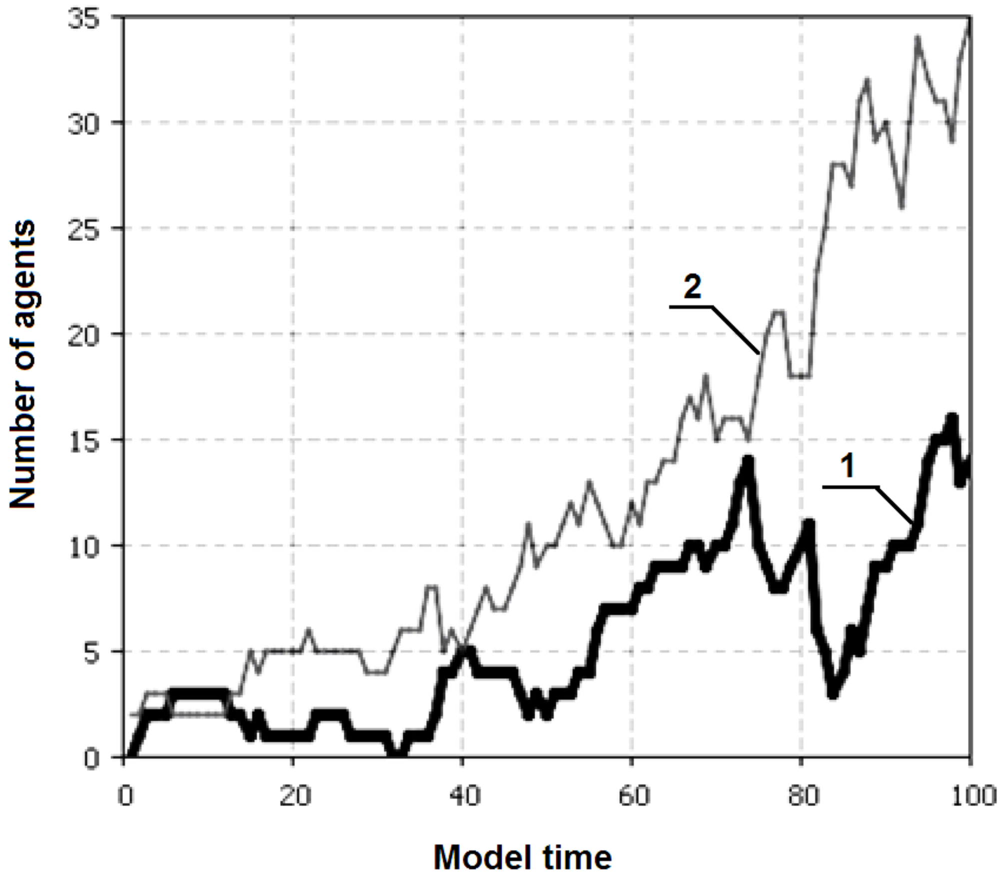


Fig.4.21. Dynamics of employment of the second-type agents under government control over the informal sector (1- employment in the shadow sector, 2- employment in the legal sector)

At the same time, illustrations presented indicate only correct qualitative response of the model to changes occurring in the sphere of employment relationships as a result of the impact of the factors considered, *i.e.* they proved only the adequacy of the proposed multi-agent model. The next step, which must be made in this direction, should be account of changes in inherited mentality and validation of the model calibrated on the basis of real statistical and demographic data.

5. Conclusion

In this book we summarized some aspects of the agent-based approach to computer simulation in economics, which uses so-called agent-dynamic modeling (ADM) [5]. ADM allows building hybrid models, where the agent communities operate in a dynamic, non-agent economic environment.

An important aspect of progress in this direction is development of efficient advanced models reflecting the behavior of these non-agent economic environments. The extended technological model (ETM) of economic system with two categories of human resources, discussed in Sec.2.2, seems to be a promising idea for representing economic environment within ADM.

Non-linear extended technological economic models (NTM), developed to imitate the dynamic non-agent economic environments, are also presented and discussed above. Different regimes of economic dynamics based on NTM, including critical phenomena arising in the simplest two-sector version of this model, which confirm the possibility of its application to real economies, are discussed. These enhanced models (see Sec.2.5) represent more realistic economic environments and could be used as counterparts along with advanced multiagent models in foundation for more sophisticated versions of ADM. We belive that NTM will became a solid foundation in the future development of ADM. Many actual economic problems can be studied on the basis of ADM using NTM.

The most relevant, in our view, directions of the future development of NTM and its applications on ADM basis, are: 1) extending of NTM on an open economy; 2) effective account of materialized technical progress and mutual influence of demographic and economic processes; and 3) development of the market-oriented NTM-based ADM, waiving global balance requirements.

In conclusion, we must note that existing well-known and emerging modern economic models reflect many aspects of socio-economic behavior [69]. Nevertheless, it should be stressed, that some important factors, which greatly affect this behavior, are still beyond the scope of computer simulations. The most important example of such situation is the influence of mentality factors on economic behavior and market relations.

The agent-based modeling seems to be the most appropriate tool to solve this problem. Understanding the role of mental features of society can significantly ease the study of corruption and oligarchy problems, inevitable in transitional economies. Some of such models are described in the last Section of this book. Despite some of them are purly multi-agents, their implementation within ADM, by combining with abive mentioned NTM into ADM hybrid model is a real perspective.

In this case such models based on ADM approach, which adequately describes the reaction of the economic system on mentality factors, must must give different resulta for different countries economies, what maybe very useful in the analysis of possible consequences of strategic decisions in transitional economies. Therefore, further development of agent-dynamic models, accounting impact of mentality factors, seems to be a promising area of research in economic modeling.

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List of abbreviations

ABM - agent-based modeling

ADM - agent-dynamic model

AETM – an aggregated extended technological model

ISB - intersectoral balance

LTM - linear technological model

NTM - non-linear technological model