EXTENDED TECHNOLOGICAL MODEL OF AN OPEN ECONOMY

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1. Introduction

Economic and social processes in Ukraine are increasingly involved in the process of globalization. Beyond the export and import relations the migration processes increased appreciably. An essential problem of our country is brain drain (the outflow of skilled personnel) and influx of unskilled labor from other countries.

The dynamics of the closed production system can be described within the framework of the von Neumann's linear technological model (LTM) [1, p.1-9]. LTM has a wide application range in economic modeling; it is able to represent practically any economic process, proceeding in such a closed system [2, p.89-93]. The extended version of LTM accounting on reproduction of human resources (HR) has been suggested recently [3, p.417-423].

The only economic system which is really closed one is the world economic system. When modeling the economic system of a selected country it is always necessary to keep in mind that it is not closed.

The aim of this work is generalization of the extended Neumann's linear technological model [3, p.417-423] on an open economy.

In the model of general economic equilibrium developed by John von Neumann [1, p.1-9] the closed economy is considered in a maximum common way as a cyclically working set of M processes, consuming and producing a finite number N of products. Thus all of products, produced in a certain period, are consumed during the following one. An enormous literature is devoted to the Neumann's model, its review one can find in the monograph of Intriligator [4, p.310-314].

2. Extended linear technological model

In the modern matrix formulation [4, p.311] of LTM for a closed economy is described by the following set of dynamic equations:

$$\mathbf{A} \cdot \boldsymbol{y}(\boldsymbol{t}) = \mathbf{B} \cdot \boldsymbol{y}(\boldsymbol{t}-1) \tag{1}$$

$$\boldsymbol{p}(t) \cdot \mathbf{B} = \boldsymbol{p}(t-1) \cdot \mathbf{A} \tag{2}$$

Here $y = \{y_1, y_2, \ldots, y_m\}^T$ is column vector of the processes intensities, and $p = \{p_1, p_2, \ldots, p_n\}$ is row vector of prices on products produced/consumed by these processes, t = 0,1,. is an index of a discrete time period, A and B are, correspondingly, (N X M) input and output matrices.

Von Neumann proved the existence of the unique solution of the above system of equations (and inequalities) of the model, which correspond to balanced growth of economy orientated on its maximum efficiency. Original Neumann's equations are more general than (1) and (2) including a set of inequalities, providing zero intensities for unprofitable processes and zero prices on the unconsumed products. LTM is a model general enough for the closed balanced economy which does not contain some obvious assumptions about character of processes and their motive forces (e.g. method of production and exchange, reasons and preferences, determining eventually the behavior of a single economic agent).

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At the same time in Neumann's LTM all consumed resources are considered as replenishable and the specific role of HR (or labor supplies, which actually are always limited) is not reflected. HR reproduced only within society in a natural non-production (noneconomic) process, and requires certain inputs for its reproduction. An attempt to introduce this additional non-production process in the Neumann's LTM, which provides the reproduction of HR, was proposed in our recent paper [3, p.417].

The general statement of problem in this work was as follows. Neumann's model was updated by adding HR as such a specific product, which is consumed by every production processes without an exception, but reproduced only in non-production process mentioned above. This additional process provides reproduction of HR, i.e. employable population, and consumes various commodities and services (educational, medical etc.), which make possible the renewal of HR and maintain their certain reproduction rate.

Intensity y_0 of non-production process and HR price (payment for unit of employed HR, wages) p_0, labeled by index 0, appear as first elements in the extended column vector of the processes intensities $\hat{y}\ (t)\ =\{y_0,\ y_1,\ y_2,\ldots,y_m\}^T$ and in the extended row vector of prices $\hat{p}(t)\ =\{p_0,\ p_1,\ p_2,\ldots,p_n\}$. The A and B matrices are expanded accordingly by zero lines and zero columns forming the augmented (N+1x M+1) $\hat{A}$ and $\hat{B}$ matrices.

Note that HR is consumed only in production processes (j=1, 2, ., M). In the non-production (j = 0) process they participate as a part of population but as a matter of fact are not utilized as HR getting no payment.

Additional elements a_{0k}, (k = 1, 2…, M) of the matrix $\hat{A}$ represent the measure of HR consumption in the production sphere, thus all of $a_{0k} > 0$, because not a single production process can operate without HR. And in the non-production process HR are not employed: $a_{00} = 0$.

Equations of the extended model after replacement of vectors y(t) and p(t) by vectors $\hat{y}\ (t)$ and $\hat{p}(t)$, and A and B matrices, accordingly by matrices $\hat{A}$ and $\hat{B}$, formally match equations (1) and (2). However the account of the HR specific features allows simplifying these equations.

Both an initial Neumann's model and its extended version [3, p.417], are applicable only to a closed economy. At the same time the real economic systems usually are open. Every company, corporation or independent country, has its own trade balance, even when exchanged by products with counterparts.

To study the economy in this case it is necessary to utilize an open model, i.e. to take into account both import and export. The model of the economic system of any country, being a model of an open system, must represent economic links with other countries. Starting from the general model of the closed world economy, we will develop below the model of an open economy of a specific country.

3. Extended technological model of an open economy

We will present the model of an open economy within the framework of the extended LTM considering the closed world economic system as consisting of two subsystems: economy of a selected country (for example, Ukraine) and the entire economy of all other countries.

The open economies of selected countries always can be considered as parts of a more general closed economic system (world economy). If the economy of the considered selected country brings in only a small contribution to the world economy, the problem of modeling such open economy is considerably simplified, because there is no need to take into account its back influence on the world economic system. For open economies of relatively small countries (small open economies), whose contribution to the world economy is not crucial, the last one serves as a distinctive «thermostat», specifying many their exogenous parameters.

Consider the world economy composed in general case from M production processes which produce N of products. In the extended model the number of these processes is increased by non-production processes reproducing HR both in the specific country under consideration and abroad. Accordingly, in the number of products must be included both domestic and foreign HR.

Many attempts to describe the open system on the base of the closed technological model have been made (see, for example, Refs. [5, p.337-383; 6, p.252-258]). The idea of extending the Neumann 's approach on the open systems has been already proposed by us before [7, p.210-215]. All the products produced in the world economic system consist of products, which are produced only in one of its subsystems and do not have analogues, and products, producible in its different subsystems, having insignificant differences in consumer properties and hence being interchangeable.

We will divide the great number of all N material products into two groups: domestic and foreign ones. Even when domestic products there are similar to the foreign ones, their prices in different countries will differ. It results, firstly, from the difference in technologies of their production, and secondly, from the presence of custom barriers. Below we will distinguish between domestic and foreign products, even if they are interchangeable. In the most general case we suppose that N = 2n, where n is a number of all physically different goods, produced in the world economic system. If a similar product is not produced in the certain subsystem, the corresponding coefficient of technological matrix B will be set to zero.

Notice that now to extend LTM we have to add not one but two kinds of products representing HR: domestic HR (labeled by index 0), and foreign HR (labeled by index n+1).

Let's denote domestic products by the index of D, and foreign ones by the index F. Accordingly we will introduce two extended n+1 - dimensional vectors: one for domestic prices: $\hat{p}_D(t)=\{p_0, p_1, p_2,\ldots, p_n\}$ and second for prices on foreign products: $\hat{p}_F(t)=\{p_{n+1}, p_{n+2},\ldots,p_{2n}\}$. Below we consider the corresponding extended 2n-dimensional 'full' row vector of prices as:

$$\hat{p}(t) =\{ \hat{p}_D(t), \hat{p}_F(t) \}. \tag{3}$$

Following the same procedure, we subdivide all M production processes into domestic (marked by index D) and foreign processes (labeled by index F).

Similarly, two additional non-production processes must be included in the extended 'full' column vector of intensities $\hat{y}(t)$: domestic non-production process with an index 0, and foreign one having index m_D+1. The full $\hat{y}(t)$ vector consists now of M+2 elements; its constituents are m_D+1-dimensional vector $\hat{y}_D(t)$ of the domestic processes intensities, and m_F+1 -dimensional vector $\hat{y}_F(t)$ of the foreign processes intensities; thus the number of production processes in the world economy remains $M = m_D + m_F$:

$$\hat{y}(t) =\{ \hat{y}_D(t), \hat{y}_F(t) \}. \tag{4}$$

Within this generalized model two technological matrices of dimension $(2n)\times(m_D+m_F)$ exist. The augmented input matrix $\hat{A}$ is:

$$\hat{A} =\begin{pmatrix} \hat{A}_D & \hat{A}_E \\ \hat{A}_1 & \hat{A}_F \end{pmatrix}, \tag{5}$$

where, $\hat{A}_X$; (X=D, T, I, F) matrices are as follows:

$\hat{A}_D$ (n+1 x m_D+1) is the consumption of domestic products and HR by domestic processes (internal consumption of products and HR);

$\hat{A}_E$ (n+1 x m_F+1) is the consumption of domestic products and HR by foreign processes (export of products and HR);

$\hat{A}_I$ (n+1 x m_D+1) is the consumption of foreign products and HR by domestic processes (import of products and HR);

$\hat{A}_F$ (n+1 x m_F+1) is the consumption of foreign products and HR by foreign processes.

Because in both non-production processes HR is not utilized as such, in the definition of Eq.(5) $\boldsymbol{a}_{00} = \boldsymbol{a}_{\boldsymbol{n},0} = \boldsymbol{a}_{0,\boldsymbol{m}_F} = \boldsymbol{a}_{\boldsymbol{m}_F,\boldsymbol{m}_F} = 0$.

According to the subdivision of products and HR given above, and taking into account that domestic products are never produced by foreign processes, and no domestic process produce foreign product, the extended technological output matrix $\hat{B}$ will look like:

$$\hat{B} = \begin{matrix} \hat{B}_D & 0 \\ 0 & \hat{B}_F \end{matrix} . \tag{6}$$

Here two matrices: $\hat{B}_D$ ($\boldsymbol{n}$+1 x $\boldsymbol{m}_D$+1) and $\hat{B}_F$ ($\boldsymbol{n}$+1 x $\boldsymbol{m}_F$+1) characterize, accordingly, outputs of domestic and foreign processes.

In addition, there are a few other zero elements in the matrix $\hat{B}$ defined by Eq.(6). HR are not produced by production processes, and non-production process does not produce material products, therefore in matrices $\hat{B}_F$ and $\hat{B}_D$ the first line and column have generally only one non-zero element, i.e. all $\boldsymbol{b}_{ij}$ except $\boldsymbol{b}_{00}$ and $\boldsymbol{b}_{\boldsymbol{m}_D+1\,\boldsymbol{n}+1}$ are zero.

In the world economy, as in any LTM of a closed economic system, two balances must be maintained 1) material balance and 2) balance of payments. To provide the description of an open economy of the selected country it is necessary to extract from these balances corresponding balances kept in the domestic economy interacting with the world economy importing and exporting products and HR.

Thus, the material balance for the general system Eq.(1) can be written down for two groups of products as follows:

$$\begin{cases} \hat{A}_D \cdot \hat{y}_D(t) + \hat{A}_E \cdot \hat{y}_F(t) = \hat{B}_D \cdot \hat{y}_D(t-1), \\ \hat{A}_1 \cdot \hat{y}_D(t) + \hat{A}_F \cdot \hat{y}_F(t) = \hat{B}_F \cdot \hat{y}_F(t-1) \end{cases} \tag{7}$$

These equations show that every product or HR, produced within any subsystem during certain period, is consumed in it or exported. So, $\hat{A}_E \cdot \hat{y}_F(t)$ is a vector of export of the selected country in terms of value, including HR migration as well; and $\hat{A}_1 \cdot \hat{y}_D(t)$ represents value of the imported products, including HR, which are utilized in the selected country during this period.

Taking into account the specific features of HR, mentioned above, it is possible to write those equations (7), which correspond to consumption of domestic HR in production processes:

$$\sum_{j=1}^{m_D} a_{0j} \cdot y_j(t) + \sum_{j=m_D+2}^{m_D+m_F+1} a_{m_D+1j} \cdot y_j(t) = b_{00} \cdot y_0(t-1) .$$

The first term in the left side indicate the consumption of domestic HR in domestic processes, and the second one the same in foreign processes (the outflow of HR from the selected country).

The balance of payments of the general system Eq.(2) also can be rewritten separately for domestic and foreign processes:

$$\begin{cases} \hat{p}_D(t)\cdot\hat{B}_D=\hat{p}_D(t-1)\cdot\hat{A}_D+\hat{p}_F(t-1)\cdot\hat{A}_I, \\ \hat{p}_F(t)\cdot\hat{B}_F=\hat{p}_D(t-1)\cdot\hat{A}_E+\hat{p}_F(t-1)\cdot\hat{A}_F \end{cases} \tag{8}$$

This system of equations shows that every domestic and foreign process defrays its own expenses, consisting of inputs of domestic and foreign products, by its own outputs.

In particular, for domestic non-production processes:

$$p_0(t)\cdot b_{00}=\sum_{i=1}^{\boldsymbol{n}} p_i(t-1)\cdot a_{i0}+\sum_{i=\boldsymbol{n}_D+2}^{2\boldsymbol{n}+2} p_i(t-1)\cdot a_{i0} \ . \tag{9}$$

This condition shows that the earnings of households must cover their consumer disbursements, consisting of expenses on domestic and imported products.

We note that unlike the domestic balance of payments, to purchase some products at the oversea markets, every country must have an available stock of foreign currency. It can be obtained only by selling the equivalent amount of own products abroad.

Supposing that the foreign trade balance of the open domestic economic system (export-import) exists in equilibrium.

Let's take into account that the selected country (we suppose this country has inconvertible domestic currency) can expend on importation in a certain period only a sum equal to value of its exports during the previous period:

$$\hat{p}_D(t)\hat{A}_E\hat{y}_F(t)=\hat{p}_F(t-1)\hat{A}_I\hat{y}_D(t-1)\,. \tag{10}$$

All the income of domestic economy consists of revenues earned inside the country and value of its exports.

Multiplying both sides of first line of Eq. (7) by vector $\hat{p}_D(t)$ from left we obtain the equation of material balance for domestic economy:

$$\hat{p}_D(t)\hat{A}_E\hat{y}_F(t)+\hat{p}_D(t)\hat{A}_D\hat{y}_D(t)=\hat{p}_D(t)\hat{B}_D\hat{y}_D(t-1)\,. \tag{11}$$

We can write down finally the full set of equations of the extended LTM of an open economy, taking now Eqs. (7-9) into consideration, as follows:

$$\begin{cases} \hat{A}_D\cdot\hat{y}_D(t)+\hat{A}_E\cdot\hat{y}_F(t)=\hat{B}_D\cdot\hat{y}_D(t-1), \\ \hat{A}_I\cdot\hat{y}_D(t)+\hat{A}_F\cdot\hat{y}_F(t)=\hat{B}_F\cdot\hat{y}_F(t-1), \\ \hat{p}_D(t)\cdot\hat{B}_D=\hat{p}_D(t-1)\cdot\hat{A}_D+\hat{p}_F(t-1)\cdot\hat{A}_I, \\ \hat{p}_F(t)\cdot\hat{B}_F=\hat{p}_D(t-1)\cdot\hat{A}_E+\hat{p}_F(t-1)\cdot\hat{A}_F. \end{cases} \tag{12}$$

We note that the solution of equations (12) must satisfy also the constraints (10).

In the case of a small open economy, first terms in second equations of (7) and (8) systems become negligible and the system of equations of the open economy (12) can be additionally simplified by taking into account only first equations from the systems (7) and (8), in which the elements of vectors $\hat{p}_F(t)$ and $\hat{y}_F(t)$ now are considered as exogenous factors:

$$\begin{cases} \hat{A}_D \cdot \hat{y}_D(t) + \hat{A}_E \cdot \hat{y}_F(t) = \hat{B}_D \cdot \hat{y}_D(t-1), \\ \hat{p}_D(t-1) \cdot \hat{A}_D + \hat{p}_F(t-1) \cdot \hat{A}_I + \hat{p}_D(t) \cdot \hat{B}_D \end{cases} \quad (13)$$

This system includes m_D+n+2 equations (m_D+1 for elements of vector $\hat{y}_D(t)$ and n+1 for elements of vector $\hat{p}_D(t)$).

Returning to the initial von Neumann's model in terms of A_D , A_D ,B_D and B_F matrices, we arrive to another form of Eq.(13), including m_D+n equations determining two vectors $\boldsymbol{y_D}(\boldsymbol{t})$ and $\boldsymbol{p_D}(\boldsymbol{t})$:

$$\begin{cases} \mathbf{A}_D \cdot \mathbf{y}_D(t) + \mathbf{A}_E \cdot \mathbf{y}_F(t) + \mathbf{C}(t) = \mathbf{B}_D \cdot \mathbf{y}_D(t-1) \\ \mathbf{p}_D(t) \cdot \mathbf{B}_D = \mathbf{p}_D(t-1) \cdot \mathbf{A}_D + \mathbf{p}_F(t-1) \cdot \mathbf{A}_I + \mathbf{D}(t-1) \end{cases}, \quad (14)$$

Besides there are two additional equations, according to extra variables y_0 and p_0:

$$\begin{cases} \sum_{k=0}^{\boldsymbol{m}_D} a_{0k} y_k(t) + \sum_{k=\boldsymbol{m}_D+1}^{\boldsymbol{M}} a_{0k} y_k(t) = b_{00} y_0(t-1) \\ p_0(t) b_{00} = \sum_{k=0}^{\boldsymbol{n}} p_k(t-1) a_{k0} + \sum_{k=\boldsymbol{n}+1}^{2\boldsymbol{n}} p_k(t-1) a_{k0} \end{cases}. \quad (15)$$

Here the elements of C and D vectors are defined as:

$$c_k(t) = a_{k0} y_0(t) + a_{k+\boldsymbol{n}\,0}\, y_{\boldsymbol{m}_D+1}(t) \quad \left(k = \overline{1,\boldsymbol{n}}\right) \quad ,$$

$$d_k(t) = p_0(t) a_{0k} + p_{\boldsymbol{n}+1}(t) a_{0\,k+\boldsymbol{m}_D} \quad \left(k = \overline{1,\boldsymbol{m}_D}\right).$$

The first equation in the system (15) actually represents a constraint on possible intensities of domestic and foreign production processes implied by available HR produced by domestic non-production process.

The second equation of the above system determines dynamics on domestic wages p_0 accounting consumption of both domestic and imported products inside the selected country.

4. Conclusions

In this work we derived the set of the Neumann's processes-product dynamic equations extended by account of HR dynamics and wages for small open economy Eqs. (14-15).

Knowing the elements of matrices $\mathbf{A}_X;\left(X = D,T,I,F\right)$ as well vectors $\hat{p}_F(t)$ and $\hat{y}_F(t)$, one can evaluate the desired vectors of prices $\hat{p}_D(t)$ and intensities $\hat{y}_D(t)$, describing economic dynamics of that small open economy within the extended LTM from Eqs.(14-15) with account of constraints (10).

The proposed modification of LTM provides the possibility to study the interrelated dynamics of economic processes and labor supplies, in particular, tracing of labor migration processes.

The approach proposed here can be developed further by adding, for example, of dutiable import, which affects the cost of imported products.

Utilizing a similar model of an open economy, it is possible also to investigate import turnover, amount and structure of export, import and its interrelation with domestic production on internal market.

Because the same product here can be treated both as product of domestic and external economy, it is possible to predict the dynamics of its price, both on domestic and world markets.

While estimating the elements of $\mathbf{A}_D$ and $\mathbf{A}_I$, $\mathbf{A}_F$ and $\mathbf{A}_E$ matrices it is possible to investigate the degree of dependence of the selected country from the certain imported products and foreign manpower, to analyze the competitiveness of its export products abroad and the demand on the domestic labor market etc.

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Summary

In this work the extension of the von Neumann's linear technological model proposed earlier is further generalized on an open economy of a small country. The proposed modification of the linear technological model provides the possibility to study the interrelated dynamics of economic processes and labor supplies, in particular, tracing of labor migration processes.

Key words: linear technological model; open economy; human resources.

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